CONCEPTUAL TOOLS

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Ref: Nilsson & Riedel, Rev 6, Prob 6.17

RLC CIRCUITS
C (CAPACITOR) EQUATIONS
Example 1

ex: $C = 0.4\,\mu F$

$$v = \begin{cases} 25V & t \le 0 \\ A_1 t e^{-1500t} + A_2 e^{-1500t} \ V & t \ge 0 \end{cases}$$

$i_c(t=0^+) = 90\,mA$ passive sign convention

a) Find initial energy stored on C.

$$w = \frac{1}{2} C v^2 = \frac{1}{2}\, 0.4\mu F\, (25V)^2 = 125\,\mu Ws = 125\,\mu J$$

b) Find A_1 and A_2. Use initial conditions:

1) $v(t=0^+) = v(t \le 0)$ since v cannot change instantly.
$= 25V$

2) $i = C\frac{dv}{dt}$ $\quad i(0^+) = 90\,mA.$

For condition (1), we plug in $t = 0^+ = 0 + \epsilon$ so small we can ignore ϵ.

$$v(t \le 0) = 25V = v(0^+) = A_1 \cdot (t=0) \cdot e^{-1500 \cdot 0} + A_2 e^{-1500 \cdot 0}\,(V)$$
$$= 0 + A_2 \cdot e^0 (V) = A_2 (V)$$

$\therefore A_2 = 25\ V$

For condition (1), we take $\frac{dv}{dt}$ for formula for $t \ge 0$

Then we plug in $t = 0^+ = 0 + \epsilon \doteq 0$ (ignore small ϵ)

$$\frac{dv}{dt} = \frac{d}{dt}\left(A_1 t e^{-1500t} + A_2 e^{-1500t}\right)$$
$$= A_1\left(e^{-1500t} + t(-1500)e^{-1500t}\right) + A_2(-1500)e^{-1500t}$$

Now plug in $t = 0$:

$$\left.\frac{dv}{dt}\right|_{t=0} = A_1\left(e^{-1500 \cdot 0} + 0(-1500)e^{-1500 \cdot 0}\right) + A_2(-1500)e^{-1500 \cdot 0}$$
$$= A_1 - 1500\,A_2 \quad V/s$$

Now use $i(t=0^+) = 90\,mA = C\left.\frac{dv}{dt}\right|_{t=0} = 0.4\,\mu F\,(A_1 - 1500 A_2)\ V/s$

We already know $A_2 = 25\ V$:

$$\therefore\ 90\,mA = 0.4\,\mu F\,(A_1 - 1500 \cdot 25)\ V/s$$

$$A_1 = \frac{90\,mA + \overbrace{0.4\,\mu F \cdot 1500 \cdot 25}^{15000\,\mu F\ V/s}}{0.4\,\mu F} = \frac{90\,mA + 15\,mA}{0.4\,\mu F}$$

$$A_1 = 0.262M\ V/s$$

or $A_1 = 262\,kV/s$

c) What is the expression for i_c?

$$i_c = C\frac{dv}{dt} = 0.4\,\mu F\left[262\,kV/s\left(e^{-1500t} - \frac{1500t}{s}e^{-1500t}\right) + 25V\left(-1500/s\right)e^{-1500t}\right]$$

$$= \left[104\,kV/s\left(1 - 1500/s \cdot t\right) - 37.5\,kV/s\right]e^{-1500t}$$

or $$i_c = \left(66.5\,kV/s - 156\,MV/s \cdot t\right)e^{-1500t}$$