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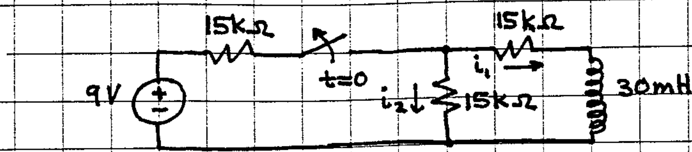
Ref: Nilsson & Riedel, Rev 6, Prob 7.5

ex:



The switch has been closed for a long time before opening at time $t=0$.

Note: Whenever a circuit configuration has existed "for a long time" it means forever — from $t=-\infty$ to $t=0$. Thus, all currents and voltages have achieved final values. In other words,
$\frac{di}{dt}=0$ for all currents, $\frac{dv}{dt}=0$ for all voltages.

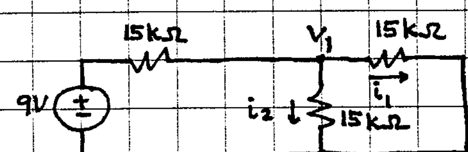
a) Find $i_1(t=0^-)$ and $i_2(t=0^-)$.

sol'n: At $t=0^-$ the circuit has had the same configuration "for a long time," and $\frac{di_1}{dt}=0$ $\frac{di_2}{dt}=0$.

For the L we have $v_L = L\frac{di}{dt} = L\cdot 0 = 0V$.

Thus, the 30mH L looks like a wire (as L's always do for their final values), and our circuit model is:

$\underline{\underline{t=0^-}}$



← L looks like a wire

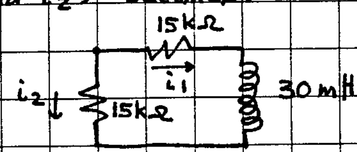
We get V_1 from V-divider: $V_1 = 9V\cdot\frac{15k\Omega \| 15k\Omega}{15k\Omega + 15k\Omega \| 15k\Omega}$

$\therefore\ V_1 = 3V$

$i_1(t=0^-) = i_2(t=0^-) = V_1/15k\Omega = \frac{3V}{15k\Omega} = 0.2\,mA$

b) Find $i_1(t=0^+)$ and $i_2(t=0^+)$.

soln: With the switch open, the 9V and 15kΩ before the switch no longer affect i_1 and i_2. Our model (for i_1 and i_2) becomes:



Since the current through the L cannot change instantly, we must have $i_1(t=0^+) = i_1(t=0^-)$.

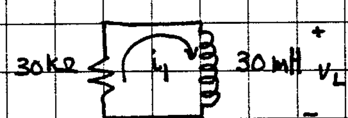
But now we also have the two 15kΩ R's in series. Thus, $i_2 = -i_1 = -i_1(t=0^-)$.

$\therefore \quad i_1(t=0^+) = 0.2\text{mA} \qquad i_2(t=0^+) = -0.2\text{ mA}$

Note: The current in an R can change instantly, as happens here (with i_2 changing sign).

c) Find $i_1(t)$ for $t \geq 0$

soln: Our circuit model (with 15kΩ R's summed) is:



$v_L = L\dfrac{di_1}{dt}$ and $v_L = -i_1 \cdot 30\text{k}\Omega$ by Ohm's law

$\therefore \quad -i_1 \cdot \underset{R}{\underset{|||}{30\text{k}\Omega}} = L\dfrac{di_1}{dt}$ or $i_1 \cdot R + L\dfrac{di_1}{dt} = 0\text{ V}$

Note: we used sum of V's around loop = 0V.

From text p. 279, $\quad i_1(t) = i(0^+)e^{-tR/L} = 0.2\text{mA}e^{-t\,30\text{k}\Omega/30\text{mH}}$

$\therefore \quad i_1(t) = 0.2\text{ mA}\cdot e^{-t\cdot 10^6/\text{s}}$

d) Find $i_2(t)$ for $t \geq 0^+$

sol'n: From part (b) we have $i_2(t) = -i_1(t)$ for $t \geq 0^+$

$$\therefore \quad i_2(t \geq 0^+) = -0.2\,\text{mA} \cdot e^{-t \cdot 10^6/5}$$

e) Explain why $i_2(0^-) \neq i_2(0^+)$.

answer: As noted in the solution to (b), the current in an R can change instantly. It does so here to keep $i_1(t)$ the same when the switch opens.

Note: The R current can change instantly because an R stores no energy. It takes time to change the energy stored in an L, and energy $= \frac{1}{2} L i^2$.