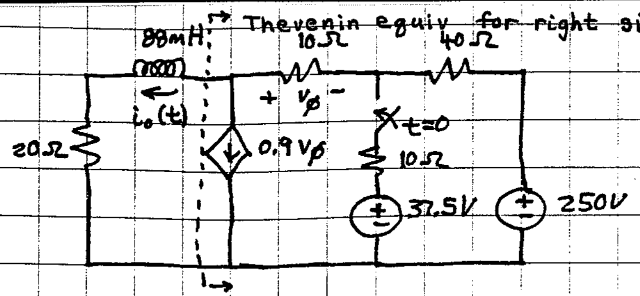
ex:

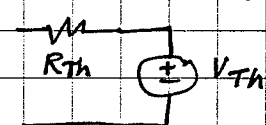


Switch has been open for a long time when it closes at $t=0$. Find $i_o(t)$, $t \geq 0$.

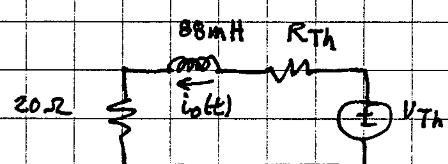
sol'n: Find $i_o(t=0^-)$. Then $i_o(t=0^+) = i_o(t=0^-)$ since current in L cannot change instantly. Use Thevenin equivalents for right side of circuit, as needed.

Qualitative sol'n: L looks like wire at $t=0^-$ since circuit has been sitting for a long time.

Thevenin equiv for right side will reduce it to:



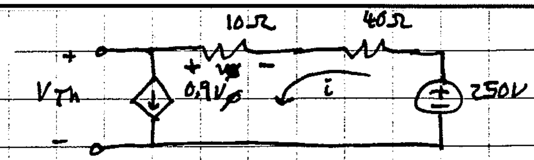
Then we will have an RL problem:



We then plug in initial $i_o(t)$ and write differential equation by summing V's around loop.

Quantitative sol'n:

Find Thevenin equiv when switch is open. First find V_{Th} for open circuit.



$i = 0.9V_\phi$ since current source is in series with 10Ω and 40Ω

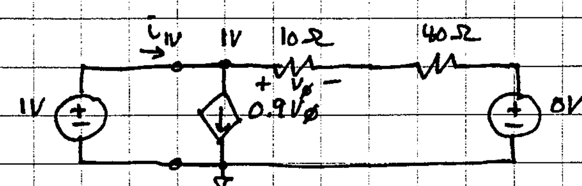
Also, $i = \dfrac{250V - V_{Th}}{10\Omega + 40\Omega}$ and $V_\phi = -10\Omega \cdot i$

$\therefore\ V_\phi = -10\Omega \cdot 0.9V_\phi = -9V_\phi$

Only solution $V_\phi = 0$. Then $i = 0$, $V_\phi = 0$, current source draws no current.

Thus, $V_{Th} = 250V$.

Now find R_{Th} by connecting $1V$ source at the two terminals on left and turning $250V$ (independent) source to $0V$.



We have $1V$ across $10\Omega + 40\Omega$. By V divider formula $V_\phi = 1V \cdot \dfrac{10\Omega}{10\Omega + 40\Omega} = 0.2V$.

Total $i_{1V} = 0.9V_\phi + \dfrac{1V}{10\Omega + 40\Omega} = 0.9 \cdot 0.2\ A + 0.02\ A$

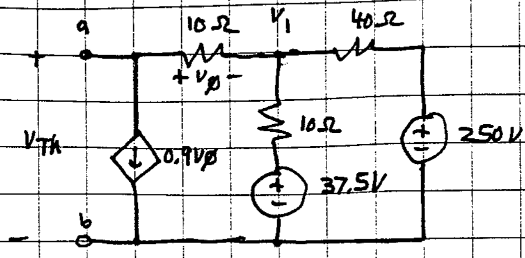
$i_{1V} = 0.2\ A$

$R_{Th} = \dfrac{1V}{i_{1V}} = 5\ \Omega$

Now $i_o(t = 0^-) = \dfrac{V_{Th}}{R_{Th} + 20\Omega}$ (see circuit with Thevenin on preceding page)

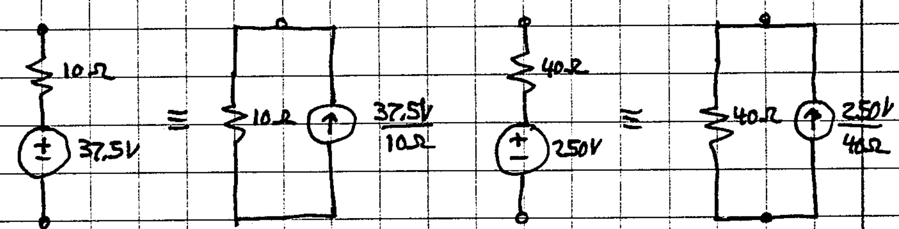
$= \dfrac{250\ V}{5 + 20\ \Omega} = \dfrac{250V}{25} = 10\ A$

Now find Thevenin equiv after switch closed.

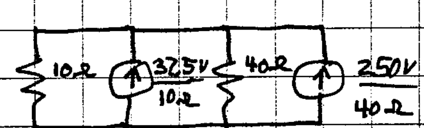


Once again we observe that $v_\phi = 0$, $0.9v_\phi = 0$ is a solution. So we just find v_1 as labeled on circuit, and we have $v_{Th} = v_1$ since $v_\phi = 0V$

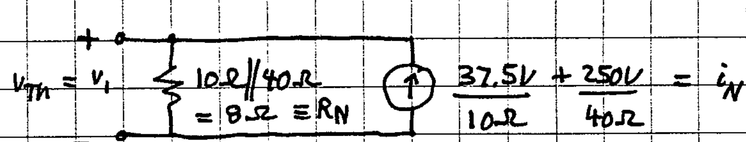
Use v-divider to find v_1, or we can use Norton equivalents:



Put the Norton equivalents together:

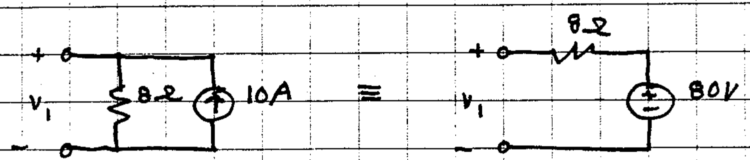


Combine current sources (add them) and use 10Ω‖40Ω.

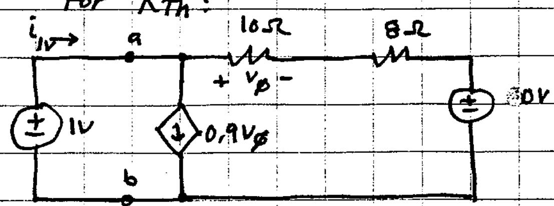


$$v_{Th} = v_1 = i_N \cdot R_N = (3.75 + 6.25)A \cdot 8\,\Omega = 80\,V$$

Now attach 1V at terminals of the Norton converted back to Thevenin equiv plus 10Ω and 0.9 v_ϕ. Then turn v_{Th} to 0V



For R_{Th}:

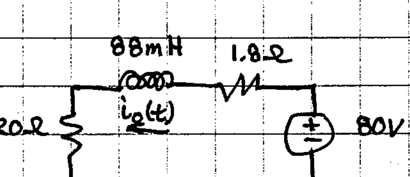


As before we have V-divider $v_\phi = 1V \cdot \frac{10\Omega}{10\Omega + 8\Omega} = \frac{10}{18} V$

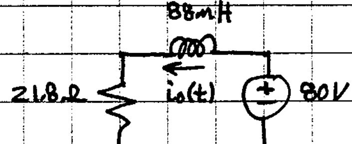
$$i_{1V} = 0.9 v_\phi + \frac{1V}{10\Omega + 8\Omega} = 0.9 \cdot \frac{10}{18} + \frac{1}{18} = \frac{10}{18} A$$

$$R_{Th} = \frac{1V}{i_{1V}} = \frac{18}{10} = 1.8\,\Omega$$

Now we can solve the RL problem:



If all we need is $i_o(t)$, we can combine R's:



$$i_o(t) \cdot 21.8\,\Omega + 88\,mH \cdot \frac{d\, i_o(t)}{dt} - 80V = 0V$$

sum V's around loop

Solution is always of form $i_o(t) = i_o(t{=}0^+) + \left(i_o(t \to \infty) - i_o(t{=}0^+)\right)\left(1 - e^{-t/L/R}\right)$

$i_o(t \to \infty) = \frac{80V}{21.8\Omega}$ (L like wire as $t \to \infty$) $i_o(t \to \infty) = 3.7\,A$

$L/R = \frac{88\,mH}{21.8\Omega} = 4.0\,ms$

$\therefore\ i_o(t) = 10 + (3.7 - 10)\left(1 - e^{-t/4ms}\right)$