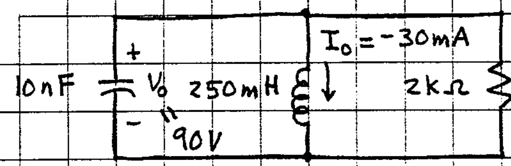

a) Find init current in each branch
b) " $v(t \geq 0)$
c) " $i_L(t \geq 0)$

ans: a) $i_R = 45$ mA
$i_L = -30$ mA
$i_C = -15$ mA

b) $v(t \geq 0) = 70e^{-10kt} + 20e^{-40kt}$

c) $i_L(t \geq 0) = -28e^{-10kt} - 2e^{-40kt}$

soln: a) $i_L(t=0) \equiv I_0 = -30$ mA (given)

$$i_R(t=0) = \frac{V_0}{R} = \frac{90V}{2k\Omega} = 45 \text{ mA}$$

$i_C = C\frac{dv}{dt}$ but this would require sol'n for $v(t)$ so we could take $\frac{dv}{dt}$. But we need $i_C(t=0)$ in order to solve differential eq'n for $v(t)$.

Use sum of currents out of node = 0:

$$i_C + i_L + i_R = 0A \qquad i_C = -(i_L + i_R)$$

or $i_C = -(-30mA + 45mA) = -15mA$

b) $v = A_1e^{s_1t} + A_2e^{s_2t}$ where s_1, s_2 are roots of characteristic eq'n.

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}, \quad \underbrace{\alpha = \frac{1}{2RC}}_{\text{damping coeff}}, \quad \underbrace{\omega_0 = \frac{1}{\sqrt{LC}}}_{\text{resonant freq}}$$

A_1, A_2 determined by matching initial conditions.

One initial condition is $v(t=0) = V_0 = 90V$.

$$v(t=0) = A_1e^{s_1 \cdot 0} + A_2e^{s_2 \cdot 0} = A_1 + A_2 = 90V.$$

Now we need another eq'n for A_1 and A_2. We must find a way of relating $\left.\frac{dv}{dt}\right|_{t=0}$ to an initial quantity known for the circuit.

This suggests that we need to use $i_C = C\frac{dv}{dt}$.

In (a) we found $i_C(t=0) = -15\text{ mA}$. Note, however, that we derived $i_C(t=0)$ from I_0, V_0. We were not given $i_C(t=0)$ directly.

$$C\left.\frac{dv(t)}{dt}\right|_{t=0} = C\left.\frac{d}{dt}\left(A_1e^{s_1t} + A_2e^{s_2t}\right)\right|_{t=0} = \left.C\left(s_1A_1e^{s_1t} + s_2A_2e^{s_2t}\right)\right|_{t=0}$$

$$" \quad = C\left(s_1A_1e^{s_1\cdot 0} + s_2A_2e^{s_2\cdot 0}\right) = C\left(s_1A_1 + s_2A_2\right) = -15\text{ mA}$$

or $C(s_1A_1 + s_2A_2) = -15\text{ mA}$

and $\boxed{A_1 + A_2 = 90\text{V}}$ (from before)

We can exploit the relationship between s_1 and s_2 to obtain a clean form of solution:

$$s_1A_1 + s_2A_2 = \left(-\alpha + \sqrt{\alpha^2 - \omega_0^2}\right)A_1 + \left(-\alpha - \sqrt{\alpha^2 - \omega_0^2}\right)A_2$$

$$= -\alpha\left(A_1 + A_2\right) + \sqrt{\alpha^2 - \omega_0^2}\left(A_1 - A_2\right)$$

We can now substitute $A_1 + A_2 = 90\text{V}$. We get:

$$C\left(s_1A_1 + s_2A_2\right) = C\left[-\alpha\underbrace{\left(A_1 + A_2\right)}_{90\text{V}} + \sqrt{\alpha^2 - \omega_0^2}\left(A_1 - A_2\right)\right] = -15\text{ mA}$$

or $\boxed{A_1 - A_2 = \dfrac{-15\text{ mA}/C + \alpha\cdot 90\text{V}}{\sqrt{\alpha^2 - \omega_0^2}}}$

Now we may add and subtract the boxed eq'ns.

Adding the boxed eq'ns gives:

$$2A_1 = 90V + \frac{-15mA/C + \alpha\, 90V}{\sqrt{\alpha^2 - \omega_0^2}}$$

Subtracting the boxed eq'ns gives:

$$2A_2 = 90V + \frac{15mA/C - \alpha\, 90V}{\sqrt{\alpha^2 - \omega_0^2}}$$

Plug in #'s now: $\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 2k \cdot 10nF} = \frac{1M}{40} = 25k/s$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{250mH \cdot 10nF} = \frac{1}{2.5n} = \frac{1G}{2.5} = 400M/s^2 = (20k/s)^2$$

$$\sqrt{\alpha^2 - \omega_0^2} = \sqrt{(25k)^2 - (20k)^2} = \sqrt{25^2 - 20^2}\ k/s$$

$$'' = \sqrt{5^2 \cdot 5^2 - 5^2 \cdot 4^2}\ k/s = \sqrt{5^2 - 4^2} \cdot 5k/s$$

$$'' = 3 \cdot 5\ k/s = 15\ k/s$$

$$15mA/C = \frac{15\ mA}{10\ nF} = 1.5\ M\ V/s$$

$$\frac{-15\ mA/C + \alpha\ 90V}{\sqrt{\alpha^2 - \omega_0^2}} = \frac{-1.5M + 25k \cdot 90}{15k} \frac{V/s}{/s}$$

$$= \frac{-1500\ k + 2250\ k}{15k}\ V$$

$$= 50V$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -25k + 15k = -10k/s \qquad s_2 = -40k/s$$

$$A_1 = \frac{90 + 50}{2}\ V = 70V \qquad A_2 = \frac{90 - 50}{2}\ V = 20V$$

$$\boxed{v(t \geq 0) = 70e^{-10kt} + 20e^{-40kt}\ V}$$

c) We obtain $i_L(t \geq 0)$ from $i_C + i_L + i_R = 0\,A$

and $i_R = \dfrac{v(t)}{R}$, $i_C = C\dfrac{dv(t)}{dt}$.

Thus, $i_L(t \geq 0) = -\dfrac{v(t)}{R} - C\dfrac{dv(t)}{dt}$

$$" = \frac{-70e^{-10kt} - 20e^{-40kt}\,V}{2k\Omega}$$

$$- 10nF\left[(-10k)\cdot 70e^{-10kt} + (-40k)\cdot 20e^{-40kt}\right]$$

$$" = -35e^{-10kt} - 10e^{-40kt}\ mA$$
$$+ 7e^{-10kt} + 8e^{-40kt}\ mA$$

$$\boxed{i_L(t \geq 0) = -28e^{-10kt} - 2e^{-40kt}\ mA}$$