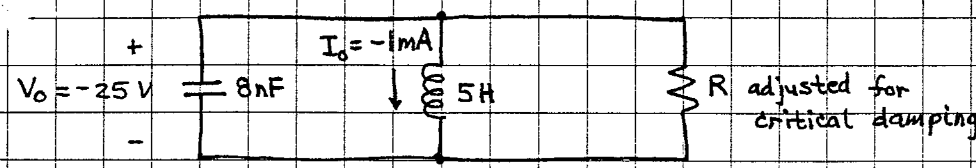


a) Calculate R

b) Calculate $v(t \geq 0)$

c) Find $v(t)$ when $i_C(t) = 0$, (i.e. at the time $i_C(t) = 0$).

d) What percentage of init stored energy remains when $i_C(t) = 0$?

ans: a) $R = 12.5\ \text{k}\Omega$

b) $v(t \geq 0) = (250\text{k}t - 25)e^{-5\text{k}t}$ V

c) $v(300\mu s) = 11.2$ V

d) % remaining $= 49.8\%$ or 50%

sol'n: a) Critical damping when $s_1 = s_2$ or $\sqrt{\alpha^2 - \omega_0^2} = 0$

or $\alpha^2 = \omega_0^2$ or $\alpha = \omega_0$ or $\dfrac{1}{2RC} = \dfrac{1}{\sqrt{LC}}$

or $R = \dfrac{\sqrt{LC}}{2C} = \dfrac{\sqrt{5H \cdot 8nF}}{2 \cdot 8nF} = \dfrac{\sqrt{40n}}{16n}$

or $R = \dfrac{1G\sqrt{40kp}}{16 \cdot} = \dfrac{1G\, 200\mu}{16 \cdot} = \dfrac{100}{8}\text{k}\Omega = 12.5\text{k}\Omega$

b) Since the circuit is critically damped our form of solution is $v(t \geq 0) = (D_1 t + D_2)e^{-s_1 t}$

where $s_1 = -\dfrac{1}{2RC} = \dfrac{-1}{2 \cdot 12.5k \cdot 8nF} = \dfrac{-1}{200\mu} = -5k/s$

and D_1, D_2 are real constants chosen so that initial circuit I_0 and V_0 are satisfied.

$$v(t=0) = (D_1 \cdot 0 + D_2)e^{-5k \cdot 0} = D_2$$

$\therefore D_2 = V_0 = -25V$ so that $v(t=0) = V_0$

Now we have to use $\left.\dfrac{dv}{dt}\right|_{t=0}$ in some way to find D_1.

Use $i_C(t=0) = C\left.\dfrac{dv(t)}{dt}\right|_{t=0}$

That means we have to find $i_C(t=0)$.

To do so, we use sum of currents out of node = 0:

$$i_C(t=0) + \underset{\displaystyle\overset{\|}{I_0}}{i_L(t=0)} + \underset{\displaystyle\overset{\|}{\frac{v(t=0)}{R}}}{i_R(t=0)} = 0A$$

or $i_C(t=0) + I_0 + \dfrac{v(t=0)}{R} = 0A$

or $i_C(t=0) + I_0 + \dfrac{V_0}{R} = 0A$

or $i_C(t=0) = -I_0 - \dfrac{V_0}{R} = --1mA - \dfrac{-25V}{12.5k} = 3mA$

Now for $C\left.\dfrac{dv(t)}{dt}\right|_{t=0}$ we have to use our symbolic sol'n:

$$C\left.\frac{dv(t)}{dt}\right|_{t=0} = C\left.\frac{d}{dt}(D_1t + D_2)e^{s_1t}\right|_{t=0} = C\left[D_1e^{s_1t} + D_1t(s_1)e^{s_1t} + D_2(s_1)e^{s_1t}\right]_{t=0}$$

Note: We have to take $\frac{d}{dt}$ first, then plug in $t=0$.

$$C\left.\frac{dv(t)}{dt}\right|_{t=0} = C\left[D_1e^{s_1\cdot 0} + D_1\cdot 0(s_1)e^{s_1\cdot 0} + D_2(s_1)e^{s_1\cdot 0}\right]$$

$$\| \quad = C\left[D_1 + 0 + D_2(s_1)\right]$$

$$\| \quad = C\left[D_1 + s_1D_2\right] = 8nF\left[D_1 + -5k/s\cdot(-25V)\right]$$

$$\therefore\ i_C(t=0) = 3mA = C\left.\frac{dv(t)}{dt}\right|_{t=0} = 8nF\left[D_1 + 5\cdot 25k\ V/s\right]$$

or $D_1 = \dfrac{3mA}{8nF} - 125k\ V/s = 375k - 125k V/s = 250\ kV/s$

$\therefore\ v(t \geq 0) = (250k/s\cdot t - 25)e^{-5k/s\cdot t}\ V$

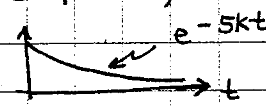
notation: units of sec^{-1} usually left out

c) $i_C(t) = C\dfrac{dv(t)}{dt} = C\left[D_1 e^{s_1 t} + D_1 t s_1 e^{s_1 t} + D_2 s_1 e^{s_1 t}\right]$

$= C\,(D_1 + D_1 t s_1 + D_2 s_1)\,e^{s_1 t}$

$i_C(t) = 0$ when $D_1 + D_1 t s_1 + D_2 s_1 = 0$, since $C, e^{s_1 t} \neq 0$.

Note: $e^{s_1 t} = e^{-5kt} \neq 0$



Substitute for D_1, D_2, s_1; rearrange eq'n:

$t = \dfrac{-(250\text{ kV/s} + -25(-5\text{k})\text{V/s})}{250\text{ kV/s}\cdot -5\text{k/s}} = \dfrac{-375}{-250\cdot 5\text{ k/s}} = +\dfrac{1.5}{5}\text{ ms}$

or $t = 0.3\text{ ms} = 300\ \mu s$

$v(300\ \mu s) = (250\text{ k/s}\cdot 300\ \mu s - 25)\,e^{-5k\cdot 300\mu}\text{ V}$

$\quad'' \quad = (250\cdot 300\text{ m} - 25)\,e^{-1500\text{m}}\text{ V} = (75-25)e^{-1.5}\text{ V}$

$v(300\mu s) = 11.2\text{V}$

d) Energy $= \frac{1}{2}Cv^2 + \frac{1}{2}Li^2$

$t=0$: $\quad w_{tot} = \frac{1}{2}\,8\text{nF}\,(-25\text{V})^2 + \frac{1}{2}\,5\text{H}\,(-1\text{mA})^2$

$= 2.5\ \mu J + 2.5\ \mu J = 5\ \mu J$

$t = 300\ \mu s$: $\quad v(300\mu s) = 11.2\text{V} \quad i_L(300\mu s) = -i_R(300\ \mu s)$
since $i_C(300\ \mu s) = 0\text{ A}$

$i_R(300\mu s) = \dfrac{v(300\mu s)}{R} = \dfrac{11.2\text{V}}{12.5\text{k}\Omega} =$

$w_{tot} = \frac{1}{2}\,8\text{nF}\cdot(11.2\text{V})^2 + \frac{1}{2}\,5\text{H}\left(-\dfrac{11.2\text{V}}{12.5\text{k}\Omega}\right)^2 = \left[4\text{n} + \dfrac{5/2}{(12.5\text{k})^2}\right](11.2)^2\text{ J}$

$= 20\text{n}\cdot 125\text{ J} = 2.5\ \mu J \quad \therefore$ % remaining $= 50\%$