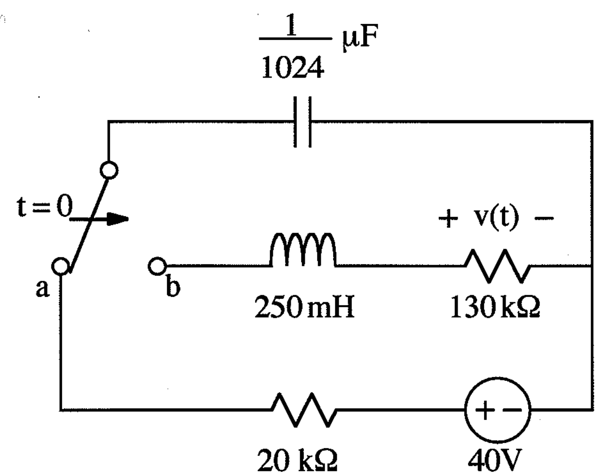
Ex:



In the above circuit, the switch moves from a to b at t = 0.

Given that no energy is stored in the inductor at time t = 0, find a numerical expression for v(t) for t > 0.

Sol'n: First, we find the initial conditions, $i_L(0^-)$ and $v_C(0^-)$.

Since there is no energy stored in the inductor and energy $w = \frac{1}{2} L i_L^2(0^-)$, we have $i_L(0^-) = 0$ A.

The capacitor will charge to 40V measured with + on the left side of C: $v_C(0^-) = 40$V.

Since the inductor current and capacitor voltage do not change instantly, we have

$$i_L(0^+) = i_L(0^-) = 0A, \quad v_C(0^+) = v_C(0^-) = 40V$$

Second, we calculate the characteristic roots of the circuit for $t > 0$.

For $t>0$, we have a series RLC circuit consisting of $R = 130\,k\Omega$, $L = 250\,mH$, and $C = \frac{1}{1024}\,\mu F$.

The characteristic roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where

$$\alpha = \frac{R}{2L} \quad \text{and} \quad \omega_0^2 = \frac{1}{LC}$$

Plugging in values, we have

$$\alpha = \frac{130\,k\Omega}{2 \cdot 250\,mH} = 260\,k \text{ rad/s}$$

$$\omega_0^2 = \frac{1}{250\,mH \cdot \frac{1}{1024}\,\mu F} = 4096\,M \text{ rad}^2/s^2$$

or $(64k \text{ rad/s})^2$

$$s_{1,2} = -260k \pm \sqrt{260^2 k^2 - 64^2 k^2} \text{ rad/s}$$

$$= -260k \pm 252k \text{ rad/s}$$

$$s_{1,2} = -8k \text{ rad/s and } -512k \text{ rad/s}$$

Third, we use the appropriate form of general solution for the over-damped case:

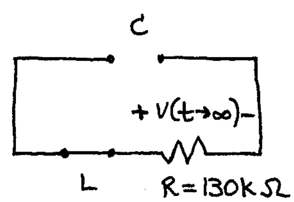
$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3$$

Fourth, we find A_3 as the final value for the circuit. As $t \to \infty$, we assume currents and voltages reach constant values. In other words, derivatives equal zero. Thus,

$$v_L = L\frac{di_L}{dt} = 0\,V \quad \text{and} \quad i_C = C\frac{dv_C}{dt} = 0\,A$$

Since $v_L = 0V$, L acts like a wire.
Since $i_C = 0A$, C acts like an open circuit.

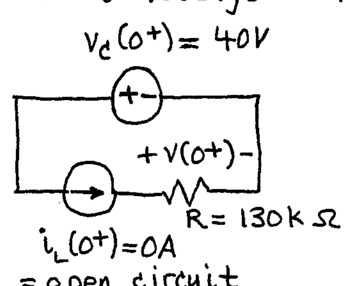
For $t \to \infty$, our circuit model is



Since C is an open circuit and there is no power source, $v(t \to \infty) = 0\,V$.

$$\therefore\ A_3 = 0\,V$$

Fifth, we find $v(t=0^+)$ by modeling L as a current source with value $i_L(t=0^+)$ and C as a voltage source with value $v_C(t=0^+)$.



Since $i_L(0^+) = 0A$, there is no current flowing in R. Thus, $v(0^+) = iR = 0V$.

$$v(0^+) = 0V$$

Sixth, we find $\left.\frac{dv(t)}{dt}\right|_{t=0^+}$ by writing $v(t)$ in terms of $i_L(t)$ and $v_C(t)$. Here,

$$v(t) = i_L(t)R.$$

$$\frac{dv(t)}{dt} = \frac{d[i_L(t)R]}{dt} = \frac{di_L(t)}{dt} \cdot R$$

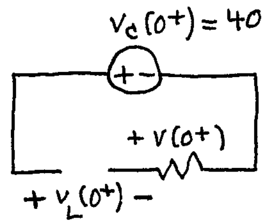
We now write $di_L(t)/dt$ in terms $v_L(t)$:

$$v_L(t) = L\frac{di_L(t)}{dt} \quad \text{or} \quad \frac{di_L(t)}{dt} = \frac{v_L(t)}{L}$$

Thus, we have

$$\frac{dv(t)}{dt} = \frac{v_L(t)}{L} \cdot R.$$

Evaluating at $t=0^+$:



We have $v_L(0^+) = v_C(0^+) = 40V$

We have

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = \frac{v_L(0^+)}{L} R = \frac{40\text{V} \cdot 130\text{k}\Omega}{250\text{mH}}$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = 20.8 \text{ MV/s}$$

Seventh, we match symbolic initial values with numerical values.

$$v(0^+) = A_1 + A_2 = 0\text{V}$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = A_1 s_1 + A_2 s_2 = 20.8 \text{ MV/s}$$

From the first eq'n, $A_2 = -A_1$.

$$A_1 s_1 - A_1 s_2 = A_1(s_1 - s_2) = 20.8 \text{ MV/s}$$

$$A_1 = \frac{20.8 \text{ MV/s}}{s_1 - s_2} = \frac{20.8 \text{ MV/s}}{-8-(-512) \text{ k r/s}}$$

$$A_1 = 41.3 \text{ V} \quad \Rightarrow \quad A_2 = -41.3 \text{ V}$$

Thus, we have our final answer:

$$v(t) = 41.3 e^{-8kt} - 41.3 e^{-512kt} \text{ V}$$