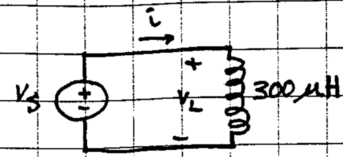
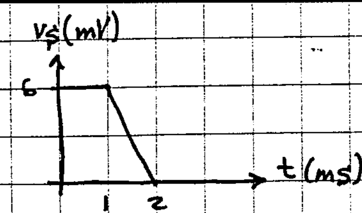
ex:





a) Find $i(t \geq 0)$ $\quad$ $i = 0$ for $t < 0$

$$v_S = v_L$$

$$v_L = L\frac{di}{dt} \qquad L = 300\,\mu H$$

$$\therefore v_S = L\frac{di}{dt}$$

But we must solve for $i(t)$ as function of v_S. This requires integration.

$$v_S\,dt = L\,di \qquad \text{(Algebra of differentials)}$$

$$\int v_S\,dt' = \int L\,di \qquad \text{Integrate both sides}$$

$$\int_{t'=0}^{t'=t} v_S(t')\,dt' = \int_{i(t'=0)}^{i(t'=t)} L\,di$$

Change name of dummy var t to t' to avoid confusion between the variable of integration, t', and actual time t.

We have added limits on the integral. Note that the limits on each integral must be of the same type as the variable of integration. Plus, the current values used as limits of integration must be for the sames as used for limits of integration on the left side.

Now we solve the integrals.
First we substitute for v_S. We have two pieces:

$$\int_{t'=0}^{t'=1ms} 6\,mV\,dt' + \underbrace{\int_{t'=1ms}^{t'=2ms} -6\frac{mV}{ms}(t-2ms)\,dt'}_{\text{Use this term only when } t > 1\text{ ms.}} = \int_{i(t'=0)}^{i(t'=t)} L\,di$$

or $\frac{6\,mV}{ms}(2ms - t)$

$$6\text{ mV}\int_{t'=0}^{t'=t} 1\cdot dt' = \int_{i(t'=0)}^{i(t'=t)} L\, di \quad \text{if } 0 \le t \le 1\text{ ms}$$

$$'' \quad = L\int_{i(t'=0)}^{i(t'=t)} 1\cdot di$$

$$6\text{ mV}\; t'\Big|_{t'=0}^{t'=t} = L\cdot i\Big|_{i(t'=0)}^{i(t'=t)}$$

$$6\text{ mV}\cdot t = L\left[i(t'=t) - i(t'=0)\right]$$

Since $i(t)$ cannot change instantly for an inductor, we have $i(t'=0) = i(t<0) = 0$ A.

We substitute also for $L = 300\,\mu H$:

$$6\text{ mV}\cdot t = 300\,\mu H\left[i(t'=t) - 0A\right]$$

$$\therefore\; i(t'=t) = \frac{6\text{mV}\cdot t}{300\,\mu H}$$

$$\text{or}\quad i(t) = 20t\,\frac{A}{s}$$

$$6\text{ mV}\int_{t'=0}^{t'=1ms} dt' + \frac{6\text{mV}}{\text{ms}}\int_{t'=1ms}^{t'=t} (2ms - t')\,dt' = \int_{i(t'=0)} L\,di \quad \text{if } 1ms \le t \le 2ms$$

$$6\text{ mV}\cdot 1ms + \frac{6\text{mV}}{\cancel{ms}}\cdot 2\cancel{ms}\,(t - 1ms)$$

$$- \frac{6\text{mV}}{ms}\,\frac{t'^2}{2}\Big|_{t'=1ms}^{t'=t} = L\; i(t) \sim \text{as before}$$

$$6\text{ mV}\left[1ms + 2\cancel{ms}\cdot(t - 1ms) - \frac{t^2}{2ms} + \frac{(1ms)^2}{2ms}\right] = 300\,\mu H\cdot i(t)$$

$$i(t) = 20\,\frac{A}{s}\left(-\frac{1ms}{2} + 2t - \frac{t^2}{2ms}\right)$$

$$= -10\text{ mA} + 40t\,\frac{A}{s} - 10\text{ mA}\left(\frac{t}{1ms}\right)^2$$

An alternative and somewhat cleaner approach is to use the final conditions of one time segment as the initial conditions for the next. We then integrate from the start of the new time segment instead of recomputing all the previous integrals.

Using this alternative approach for $1\,ms \le t \le 2\,ms$, we have initial conditions $i(t=1\,ms) = 20 \cdot 1\,ms \frac{A}{s} = 20\,mA$ from the final time, $t = 1\,ms$, for the solution for $0 \le t \le 1\,ms$.

Note the limits we now use on our integrals:

$$6 \frac{mV}{ms} \int_{t'=1\,ms}^{t'=t} (2\,ms - t')\, dt' = \int_{\underbrace{i(t'=1\,ms)}_{20\,mA}}^{i(t'=t)} L\, di \quad \text{if } \underline{\underline{1\,ms \le t \le 2\,ms}}$$

$$\frac{6\,mV}{300\,\mu H} \int_{t'=1\,ms}^{t'=t} \left(2 - \frac{t'}{ms}\right) dt' = i(t) - 20\,mA$$

$$20 \frac{A}{s} \left[2(t - 1\,ms) - \frac{t^2 - (1\,ms)^2}{2\,ms} \right] = i(t) - 20\,mA$$

$$20\,mA \left[2\left(\frac{t}{ms} - 1\right) - \frac{t^2 - (1\,ms)^2}{2(ms)^2} \right] + 20\,mA = i(t)$$

$$i(t) = -10\,mA + 40\,mA \cdot \frac{t}{ms} - 10\,mA \frac{t^2}{(ms)^2}$$

Same answer, but less work.

For $2\,ms \le t$, we have $v_s = 0$ so $L\frac{di}{dt} = 0$.

Thus, $i(t)$ stays constant at the final value of $i(t)$ for $t = 2\,ms$ given by the equation for $i(t)$ when $1\,ms \le t \le 2\,ms$.

$$i(t \ge 2\,ms) = -10\,mA + 40\,mA \cdot \frac{2\,ms}{ms} - 10\,mA \frac{(2\,ms)^2}{(ms)^2} = 30\,mA.$$