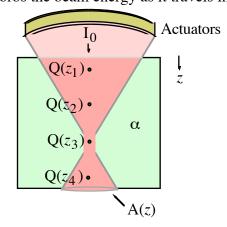
**OPTIMIZATION** NONLINEAR REGRESSION Example problem 1

**Ex:** An ultrasound beam of intensity  $I_0$  is created by actuators arranged in a crescent, as shown in the figure below. The beam travels through an object (shown in green) that absorbs the beam energy as it travels in the *z* direction.



The following equation, derived from physics, describes the intensity that sensors measure as a function of distance, z, into the medium.

$$Q(z) = I_0 \frac{\alpha e^{-2\alpha z}}{A(z)}$$

where

z = distance into object

 $I_0 = initial beam intensity$ 

Q(z) = measured intensity at z

A(z) = beam area at z

 $\alpha = \text{coefficient of absorption for object}$ 

The optimization problem is to derive a value for  $\alpha$  from measured values of intensity,  $Q(z_1)$ ,  $Q(z_2)$ ,  $Q(z_3)$ , and  $Q(z_4)$ , versus distance z into the object. The beam area versus distance, z, is known from geometry (plus an approximation at  $z_3$  where the beam is focused but must have an effective area greater than zero):

$$z_1 = 1$$
,  $z_2 = 2$ ,  $z_3 = 3$ ,  $z_4 = 4$   
A( $z_1$ ) = 8, A( $z_2$ ) = 2, A( $z_3$ ) = 1, A( $z_4$ ) = 2

One approach is to find the  $\alpha$  that minimizes the following squared error function:

$$E = \sum_{i=1}^{4} [Q(z_i) - Q(z)]^2$$