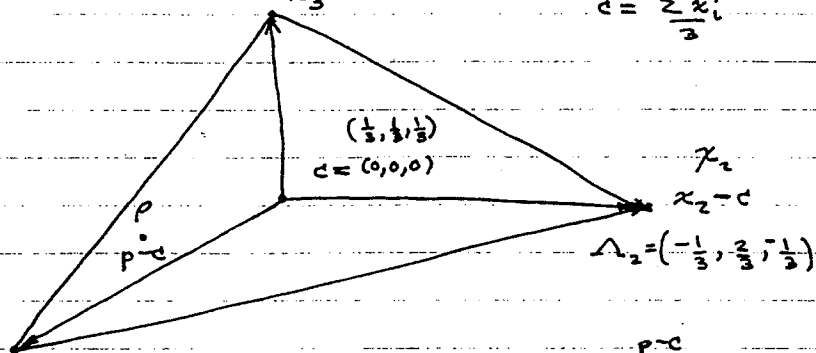


Cubic Splines - N-Dimensions -  $\chi_3$

il Potter  
Aug 1993

Calculation of  $\lambda$ 's,  $\Lambda_3 = \begin{pmatrix} -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \end{pmatrix}$

$$c = \frac{\sum x_i}{3}$$



$$\Lambda_1 = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$\begin{bmatrix} -x_1 - \\ -x_2 - \\ -x_3 - \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

$$\vec{a} = \chi^{-1} \vec{p}$$

$$\lambda_1 = a_1 \Lambda_{11} + a_2 \Lambda_{21} + \frac{1}{3} + \frac{2}{3}$$

$$\vec{\lambda} = \Lambda \vec{a} + \frac{1}{3} \vec{1}$$

$$\lambda_2 = a_1 \Lambda_{12} + a_2 \Lambda_{22} + \frac{1}{3} + \frac{2}{3}$$

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix}$$

$$\lambda_3 = a_1 \Lambda_{13} + a_2 \Lambda_{23} + \frac{1}{3} + \frac{2}{3}$$

ex:  $p = x_1$        $a_1 = 1$        $a_2 = 0$

$$\lambda_1 = 1 \cdot \frac{2}{3} + 0 \cdot \frac{-1}{3} + \frac{1}{3} + \frac{2}{3} = 1 \quad \checkmark$$

$$\lambda_2 = 1 \cdot \frac{-1}{3} + 0 \cdot \frac{2}{3} + \frac{1}{3} = 0 \quad \checkmark$$

$$\lambda_3 = 1 \cdot \frac{-1}{3} + 0 \cdot \frac{-1}{3} + \frac{1}{3} = 0 \quad \checkmark$$

In general  $\vec{\Lambda}_i = (0, \dots, 0, 1, 0, \dots, 0) - \frac{1}{n} (1, 1, \dots, 1)$

$$\vec{a} = \chi^{-1} \vec{p} \quad \vec{x}_i = \vec{x}_i - \vec{c} \quad \vec{c} = \frac{1}{n} \sum \vec{x}_i$$

$$\vec{\lambda} = \Lambda \vec{a} + \frac{1}{n} \vec{1} \quad \chi = \begin{bmatrix} -x_1 - \\ \vdots \\ -x_{n-1} - \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \Lambda_1 & \dots & \Lambda_{n-1} \\ \vdots & & \vdots \end{bmatrix}$$

Should always be well conditioned