TUTORIAL: STATE VARIABLES and MATLAB®

Time-domain analysis of circuits with more than one L and C is difficult because it requires solution of characteristic equations higher than second degree. This supplement illustrates the use of MATLAB® functions, ode23 and ode45, for solving a system of coupled first-order differential equations of the form

$$d\mathbf{x}/dt = f(\mathbf{x},t)$$

where $\mathbf{x}$ is a vector called the *state vector,* and t is the independent variable, which in our case will be time. As indicated in *Mastering MATLAB® 7*, Chapter 25, a differential equation can always be expressed as a system of coupled first-order differential equations. The MATLAB® functions are powerful because they can be used to solve nonlinear as well as linear differential equations.

Consider the third-order circuit in Fig. 1 as an example for illustrating the use of the state vector (state variables) and ode45. Let's use MATLAB® to plot v_2 as a function of time. Using Kirchhoff's laws in the time domain, we could write the third-order differential equation for v_2 and then convert it to three coupled first-order equations following the procedure outlined in the MATLAB® book. It is much easier, however, to obtain the system of coupled first-order equations directly in terms of what are called the *state variables*. For circuits, the state variables are the currents through inductances and the voltages across capacitances. Using Kirchhoff's laws in the time domain, it is easy to write first-order differential equations in terms of these variables because $i = C\,dv/dt$ for a capacitance, and $v = L\,di/dt$ for an inductance, both of which involve first derivatives.

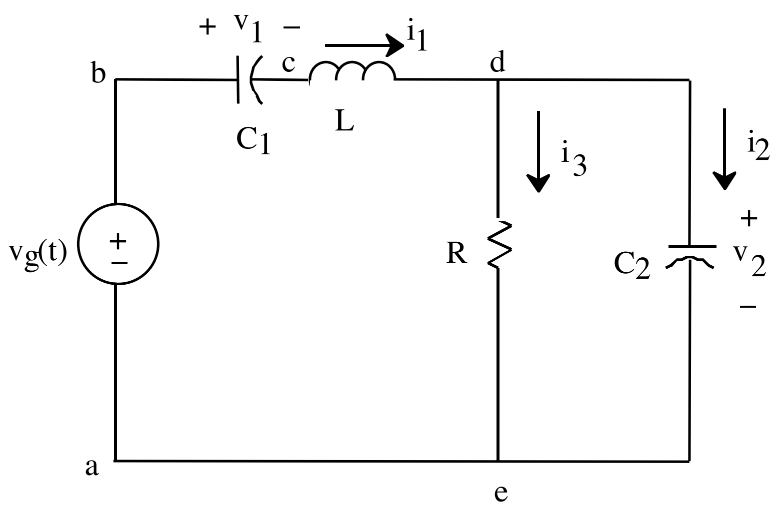

Fig. 1. Third-order circuit. $v_g(t)$ is a step function that switches from $-v_0$ to v_0 at $t = 0$.

The state variables for the circuit in Fig. 1 are v_1, v_2, and i_1. The first equation is easily obtained from the relationship between the voltage and current for C_1:

(1) $dv_1/dt = i_1/C$

The second equation is obtained by writing Kirchhoff's voltage equation around the closed path *abcdea* :

$$-v_g + v_1 + L\, di_1/dt + v_2 = 0$$

from which we get

(2) $di_1/dt = -\, v_1/L - v_2/L + v_g/L$

The third equation is obtained by using Kirchhoff's current law,

$$i_1 = i_2 + i_3$$

and $i_3 = v_2/R$, $i_2 = C_2\, dv_2/dt$ to get

(3) $dv_2/dt = i_1/C_2 - v_2/RC_2$

Now defining the state vector (note that x is a column vector) as

(4) $x = [\ v_1\ i_1\ v_2]'$

we get the three coupled first-order differential equations from (1), (2), (3), and (4):

(5) $dx_1/dt = x_2/C_1$

(6) $dx_2/dt = - x_1/L - x_3/L + v_g/L$

(7) $dx_3/dt = x_2/C_2 - x_3/RC_2$

Before writing the Matlab® program, we need the initial conditions on the vector x. The initial conditions for x are much easier to obtain than for the third-order differential equation for v2. Because the current cannot change instantaneously through an inductance, and because the voltage across a capacitance cannot change instantaneously,

$$v_1(0+) = v_1(0-) = -v_0$$

$$i_1(0+) = i_1(0-) = 0$$

$$v_2(0+) = v_2(0-) = 0$$

The initial value of x is thus

(8) $x0 = [-v_0 \ \ 0 \ \ 0]'$

The ode45 function requires a function that calculates `xdot` (which stands for dx/dt) as a function of `t` and `x` (see MATLAB® book). Here is a function that calculates `xdot` as described by (5)-(7):

```
function xdot = ckt(t,x)
vg=1;
C1=100e-9;
C2=100e-9;
R=30;
L=184e-6;
xdot(1) = x(2)/C1;
xdot(2) = -(x(1) + x(3))/L + vg/L;
xdot(3) = x(2)/C2 - x(3)/(R*C2);
xdot = [xdot(1); xdot(2); xdot(3)];
```

To solve for **x** and plot v_2, we can now type the following in the command window:

```
t0=0; tf=60e-6;
tspan=[t0,tf];
x0=[-1 0 0]' ;
[t,x]=ode45('ckt',tspan,x0);
plot(t,x(:,3))
```

We could also plot all three components of **x** by using `plot(t,x)`, as shown in Fig. 2.

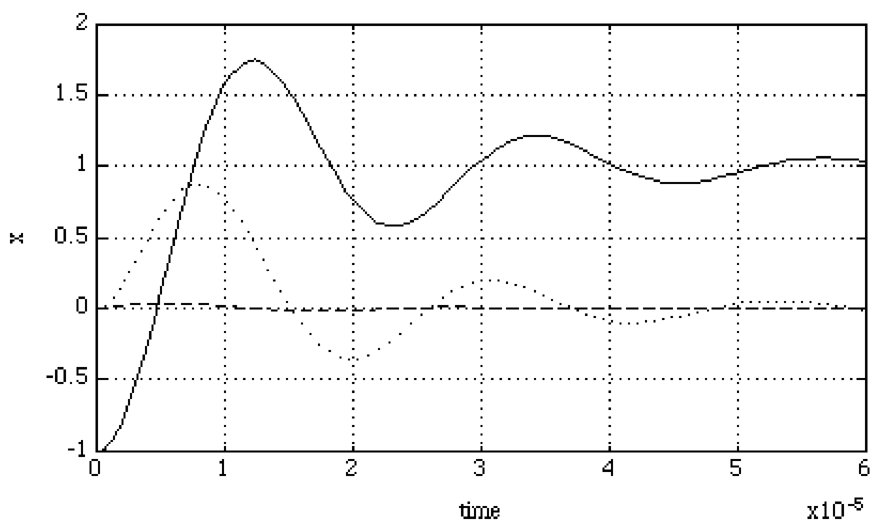

Fig. 2. Graph of the state vector x versus time for the circuit of Fig. 1.

If `tf` is chosen to be too large, the matrix size limitation in the student edition of MATLAB® will be exceeded. Also, sometimes choosing `tf` to be too large will cause a message "singularity expected," and the function will not be executed.

EX: Use ode45 to solve for x for the circuit in Fig. 3 when R1 = 20 Ω, R2 = 5 Ω, C1 = 100 nF, C2 = 200 nF, L = 300 μH, vo = 1 V. Then plot `i1` versus `t` and `i3` versus `t`.

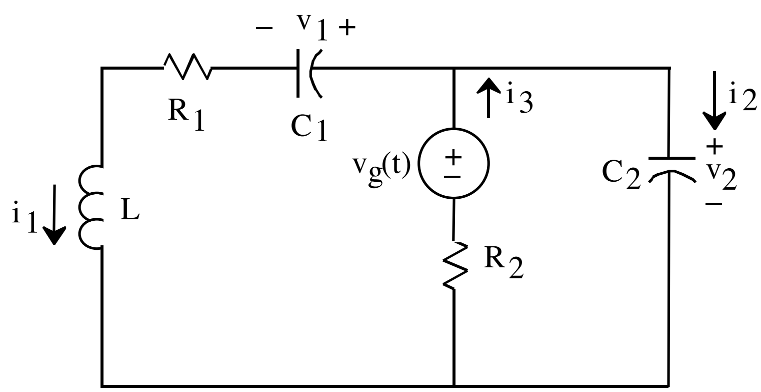

Fig. 3. The voltage $v_g(t)$ changes instantly from $-v_o$ to v_o at $t = 0$.

GIVEN:

$$x = [i_1 \;\; v_1 \;\; v_2]'$$

$$dx_1/dt = (x_3 - x_2)/L - x_1 R_1/L$$

$$dx_2/dt = x_1/C_1$$

$$dx_3/dt = -x_1/C_2 + (v_g - x_3)/(R_2 C_2)$$

$$x_o = [0 \;\; -v_o \;\; -v_o]'$$

ANS: