EX: Using a spreadsheet or numerical program with observations drawn from a standard gaussian (normal) distribution, (i.e., $\mu = 0$ and $\sigma^2 = 1$), calculate the estimated standard deviation, $\hat{\sigma}$, 15 times based on ranges of 12 samples of 20 observations each. Also, calculate the sample mean and sample standard deviation for these 15 estimated standard deviations.

SOL'N: We use a Matlab® program, (see StatsControlChartXbarEx1.m file), to perform calculations based on the following equations from [1]:

$$R_i = x_{i,\max} - x_{i,\min} \qquad \overline{R} = \frac{1}{k}\sum_{i=1}^{k} R_i \qquad \hat{\sigma} = \frac{\overline{R}}{d_2}$$

where k ≡ total number of samples being considered
i ≡ index designating sample
x_{ij} ≡ observations available for sample i
d_2 ≡ number of observations in each sample
$\hat{\sigma}$ ≡ estimated value of σ

Results for one run of the program are as follows:

$\hat{\sigma}_1 = 0.969$ $\hat{\sigma}_2 = 1.032$ $\hat{\sigma}_3 = 1.057$ $\hat{\sigma}_4 = 0.974$ $\hat{\sigma}_5 = 1.072$

$\hat{\sigma}_6 = 1.084$ $\hat{\sigma}_7 = 1.044$ $\hat{\sigma}_8 = 1.102$ $\hat{\sigma}_9 = 1.047$ $\hat{\sigma}_{10} = 0.982$

$\hat{\sigma}_{11} = 1.025$ $\hat{\sigma}_{12} = 1.026$ $\hat{\sigma}_{13} = 1.028$ $\hat{\sigma}_{14} = 1.016$ $\hat{\sigma}_{15} = 0.994$

The calculated mean of the calculated $\hat{\sigma}$'s is within a few percent of the true σ, and the standard deviation of the calculated $\hat{\sigma}$'s is only a few percent of the true σ:

$$\bar{x}_{\hat{\sigma}} = 1.0302$$

$$s_{\hat{\sigma}} = 0.0393$$

REF: [1] Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, and Keying Ye, *Probability and Statistics for Engineers and Scientists,* 8th Ed., Upper Saddle River, NJ: Prentice Hall, 2007.