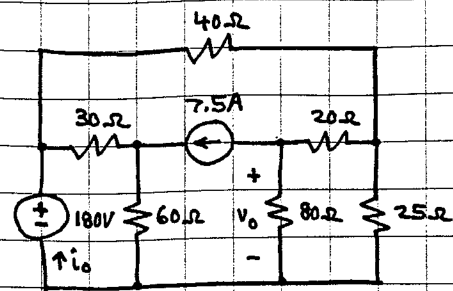
Ref: Nilsson & Riedel, Rev 6, Prob 6.17

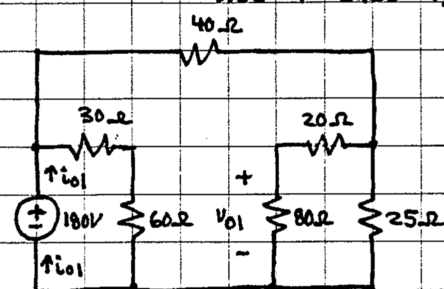
ex:



Use superposition to solve for i_o and v_o in the circuit.

soln: We turn on one v- or i-source at a time, find i's and v's, and then sum results from each circuit to find total i's and v's.

Circuit 1: 180V source on, 7.5A source set to 0A (open circuit).
Use '1' subscripts.



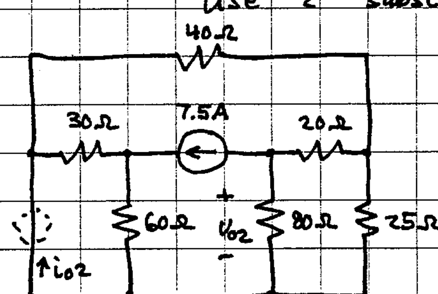
$$i_{o1} = \underbrace{\frac{180V}{30+60\,\Omega}}_{2A} + \underbrace{\frac{180V}{40+(80\|25\,\Omega+20)}}_{3A} = 5A$$

$$v_{o1} = 180V \frac{(20+80)\|25\,\Omega}{40+(80\|25\,\Omega+20)} \cdot \frac{80\,\Omega}{20\,\Omega+80\,\Omega} = 48V$$

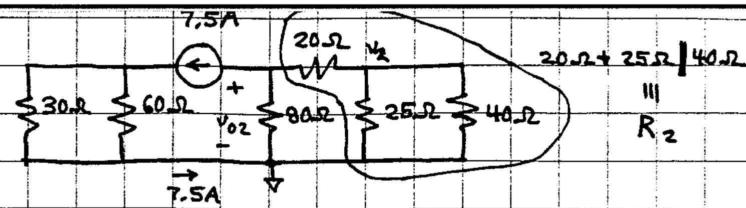
↑ v-divider, can ignore 30Ω, 60Ω

or use $v_{o1} = 80\,\Omega \cdot 3A \cdot \frac{25\,\Omega}{20+80+25\,\Omega}$ i-divider

Circuit 2: 180V source off, 7.5A source on
(= 0V = wire)
Use '2' subscripts



Note that the 40Ω R is actually in parallel with the 25Ω R on the right. We can find v_{o2} (but not i_{o2}) with the following circuit:



Now we have an i-divider on the right.

$$i_{80\Omega} = -7.5A \cdot \frac{R_2}{R_2 + 80\Omega} \qquad v_{o2} = i_{80\Omega} \cdot 80\Omega$$

$$v_{o2} = -7.5A \cdot \frac{R_2 \cdot 80\Omega}{R_2 + 80\Omega} = -7.5A \cdot R_2 \| 80\Omega \text{ (of course!)}$$

$$R_2 = 20 + 5\Omega \cdot 5\|8 = 20 + 5 \cdot \frac{40}{13} = \frac{460}{13}$$

$$R_2 \| 80\Omega = \frac{460 \cdot 80}{13 \cdot \left(\frac{460}{13} + 80\right)} = \frac{460 \cdot 8}{46 + 13 \cdot 8} = \frac{3680}{150} = \frac{368}{15}$$

$$v_{o2} = -7.5 \cdot \frac{368}{15} = -\frac{368}{2} = -184 \text{ V}$$

Now for i_{o2}. We can find v_2 shown on above circuit diagram. Then this will give us the current in the 40Ω R. This will also give us the current through the 80Ω and 25Ω R's.

We also observe that the 30Ω and 60Ω R's form a current divider.

$$i_{30\Omega} = 7.5A \cdot \frac{60\Omega}{30\Omega + 60\Omega} = 5A$$

$$i_{60\Omega} = 7.5A \cdot \frac{30\Omega}{30\Omega + 60\Omega} = 2.5A$$

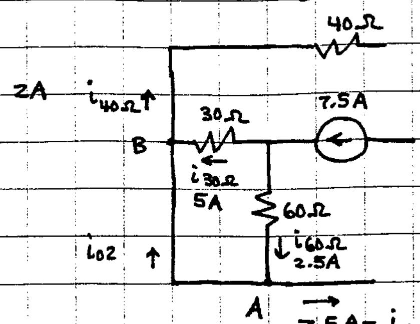
We find v_2 from V-divider:

$$v_2 = v_{o2} \cdot \frac{25\|40\Omega}{20\Omega + 25\|40\Omega} = -184V \cdot \left(\frac{\frac{200}{13}}{\frac{460}{13}} = \frac{200}{460} = \frac{10}{23} \right)$$

$$v_2 = -80V$$

$$i_{40\Omega} = \frac{80V}{40\Omega} = 2A$$

Now back to the original circuit #2 for a detailed look at i_{o2}:



$\overrightarrow{7.5A - i_{40\Omega} = 5.5A}$

↑ This current + $i_{40\Omega}$ must equal 7.5A from source.

$i_{o2} = i_{60\Omega} - 5.5A = 2.5A - 5.5A = -3A$

from sum of currents out of node A = 0A.

check $i_{o2} = i_{40\Omega} - i_{30\Omega} = 2A - 5A = -3A$ ✓

from sum of currents out of node B = 0A.

Now add results from circuits 1 and 2:

$v_o = v_{o1} + v_{o2} = 48V + -184V = -136V$

$i_o = i_{o1} + i_{o2} = 5A + -3A = 2A.$