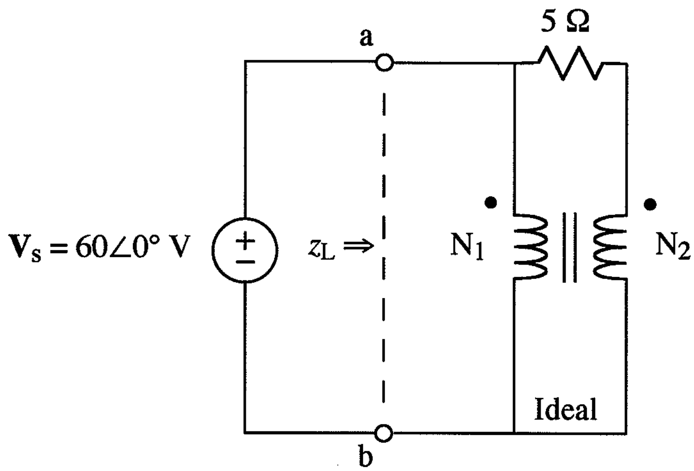
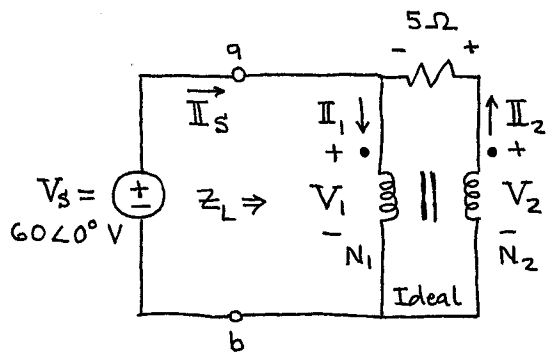
Ex:



Given $N_1/N_2 = 9$, calculate the impedance, z_L, seen by the voltage source in the above transformer circuit.

Sol'n: We begin by labeling the transformer V's and I's.



Note: The + signs for V_1 and V_2 are at the dots. I_1 flows into the dot, and I_2 flows out of the dot.

Now we use Ohm's law to find z_L as $z_L = V_S / I_S$.

From a current summation at the node to the left of the 5Ω resistor, we have

$$-I_S + I_1 - I_2 = 0\,A. \qquad (1)$$

Since one node is always redundant for current summations, this is the only current summation equation for the circuit.

We do, however, have that I_2 flows thru the 5Ω resistor, giving a voltage drop with the polarity indicated on the above circuit diagram.

This V-drop is part of the voltage loop on the right side which yields the following eq'n:

$$V_1 + I_2 \cdot 5\Omega - V_2 = 0\,V \qquad (2)$$

We also have the voltage loop on the left:

$$V_S - V_1 = 0\,V \quad \text{or} \quad V_S = V_1 \qquad (3)$$

Now we use the ideal transformer eq'ns:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Eq'n (2) becomes (also using $V_1 = V_S$)

$$\mathbb{I}_2 \cdot 5\Omega = V_2 - V_1$$

$$" = V_1 \frac{N_2}{N_1} - V_1$$

$$\mathbb{I}_2 \cdot 5\Omega = V_1 \left(\frac{N_2}{N_1} - 1 \right)$$

$$\mathbb{I}_2 = \frac{V_1 \left(\frac{N_2}{N_1} - 1 \right)}{5\Omega} = \frac{V_S \left(\frac{N_2}{N_1} - 1 \right)}{5\Omega}$$

$$\mathbb{I}_2 = \frac{60\angle 0^\circ V \cdot \left(\frac{1}{9} - 1 \right)}{5\Omega}$$

$$= 12\angle 0^\circ A \cdot \left(-\frac{8}{9} \right)$$

$$\mathbb{I}_2 = -\frac{32}{3}\angle 0^\circ A = -10.67\angle 0^\circ A$$

Eq'n (1) becomes

$$\mathbb{I}_S = \mathbb{I}_1 - \mathbb{I}_2$$

$$= \mathbb{I}_2 \frac{N_2}{N_1} - \mathbb{I}_2$$

$$= \mathbb{I}_2 \left(\frac{N_2}{N_1} - 1 \right)$$

$$= \mathbb{I}_2 \left(\frac{1}{9} - 1 \right)$$

$$\mathbb{I}_S = \mathbb{I}_2 \left(-\frac{8}{9} \right)$$

Now we compute $z_L = \dfrac{V_S}{I_S}$.

$$z_L = \frac{V_S}{I_S} = \frac{V_S}{I_2\left(-\frac{8}{9}\right)} = \frac{\cancel{V_S}}{\dfrac{\cancel{V_S}\left(\frac{N_2}{N_1}-1\right)}{5\,\Omega}\underbrace{\left(\frac{N_2}{N_1}-1\right)}_{-\frac{8}{9}}}$$

$$z_L = \frac{5\,\Omega}{\left(\frac{N_2}{N_1}-1\right)^2} = \frac{5\,\Omega}{\left(-\frac{8}{9}\right)^2}$$

$$z_L = 5\,\Omega \cdot \frac{9^2}{8^2} = 5\,\Omega \cdot \frac{81}{64}$$

$$z_L = 6.3\,\Omega$$