INTRO: In WWII, British intelligence intercepted and decoded messages sent by the Germans using the "Enigma" machine. Alan Turing lead the effort, which lead to the creation of early computers and is portrayed in the movie "Imitation Game".

Today, encryption has become more complicated. One interesting scheme to openly publish a public key the may be used to encode messages. The decoding requires a private key, which could be found by anyone with enough computing power. The time required to find the private key, however, is so large as to be impractical.

Today, you will break codes based on factoring polynomials. The code is simple enough to break, *if* you can factor cubic polynomials.

CODE: You will have the following information for each code you break, (example values in 3rd line):

x^3 Coefficient	a_2 Coefficient	a_1 Coefficient	a_0 Coefficient	p = public root
a_3	a_2	a_1	a_0	p
1	13	54	72	3

The first four numbers are the coefficients of a cubic polynomial:

$$f(x) = x^3 + 13x^2 + 54x + 72 \tag{1}$$

The public root means one root term is $(x + p) = (x + 3)$. The information being sent by the code is the other two root terms. The size of the largest root (lives saved) is the value of the code. You will earn points (or cash) proportional to the largest root.

Continuing with our example, we factor out the $(x + 3)$ term from $f(x)$.

$$f(x) = (ax^2 + bx + c)(x + 3) = x^3 + 13x^2 + 54x + 72 \tag{2}$$

It turns out the factorization is straightforward:

$$a = 1 \qquad b = a_2 - p = 13 - 3 = 10 \qquad c = \frac{a_0}{p} = \frac{72}{3} = 24 \tag{3}$$

After you calculate b and c, you will factor the quadratic function: $x^2 + bx + c$.

$$x^2 + bx + c = x^2 + 10x + 24 = (x + d_1)(x + d_2) = (x + 4)(x + 6) \tag{9}$$

The value of this broken code is the larger-sized root term: 6.

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