CIRCUIT: The circuit shown below in Fig. 1 produces a rectangular wave with frequency controlled by potentiometer, R_a, R_b. The duty cycle varies with v_{ctrl}. For an op-amp such as the LM324 with asymmetric rail voltages, the duty cycle will be greater than 50% when the control voltage is at reference. A control voltage halfway between the rail voltages, however, produces a 50% duty cycle waveform.

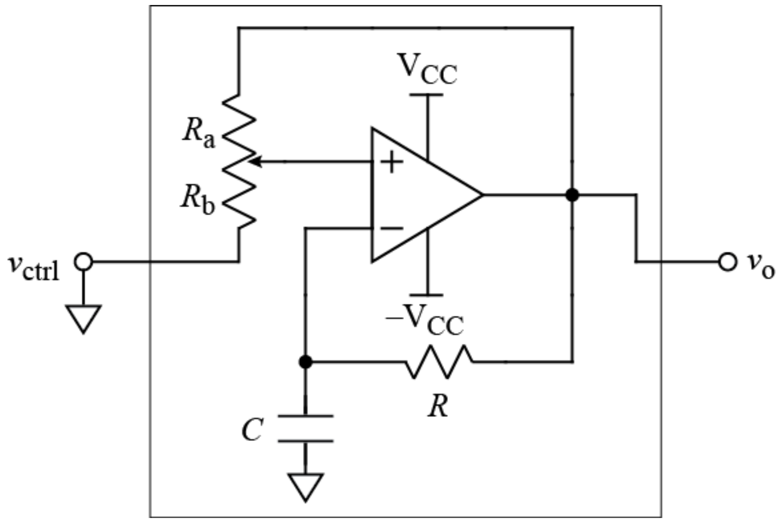

Fig. 1. Oscillator.

Fig. 2 shows the waveforms for the oscillator.

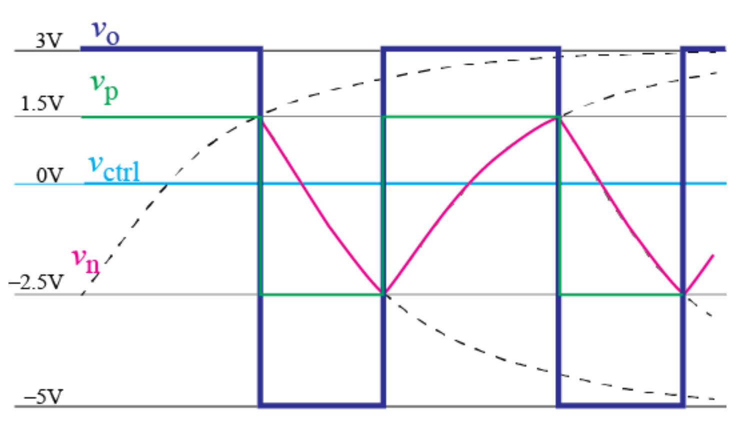

Fig. 2. Oscillator waveforms for LM324 op-amp [1] with ±5V supplies.

When the output goes high, the capacitor starts charging toward the positive rail voltage. The positive rail voltage, along with potentiometer, R_a, R_b, and the control voltage, v_{ctrl}, create a voltage divider that determines how high the output voltage, v_o, rises before the op-amp, acting as a comparator, switches to negative rail voltage output. The same voltage divider is now fed by a negative voltage that determines how low the output voltage, v_o, drops before the op-amp, acting as a comparator, switches to positive rail voltage output. The cycle then repeats.

Analysis of circuit:

To determine the timing of the output waveform, we solve *RC* charging problems for the rising capacitor voltage. The solution for falling capacitor voltage is obtained by switching the $v_{+\text{rail}}$ and $v_{-\text{rail}}$ and inverting the value of v_{ctrl}.

The initial voltage for the *RC* charging problems is the trip point, v_p in Fig. 2, determined by the voltage divider fed by $v_o = v_{-\text{rail}}$ and v_{ctrl}.

$$v_C(0^-) = v_p(0^-) = \frac{v_{-\text{rail}}R_b + v_{\text{ctrl}}R_a}{R_a + R_b}$$

The final destination voltage is the positive rail voltage for v_o, although switching occurs before this voltage is reached.

$$v_C(t \to \infty) = v_{+\text{rail}}$$

The time constant, *RC*, primarily determines the oscillation frequency, whereas v_{ctrl} primarily controls the duty cycle.

$$\tau = RC$$

It is recommended that the duty cycle be set first with v_{ctrl}. Then *R* may be adjusted to set the oscillation frequency.

The equation for the charging and discharging curves:

$$v_C(t) = \left[v_C(0^-) - v_C(t \to \infty)\right]e^{-t/\tau} + v_C(t \to \infty).$$

Solving for the time of a half-cycle, we set the trip point equal to the capacitor voltage:

$$v_C(t) = v_p = \frac{v_{+\text{rail}}R_b + v_{\text{ctrl}}R_a}{R_a + R_b} = \left[v_C(0^-) - v_C(t \to \infty)\right]e^{-t/\tau} + v_C(t \to \infty)$$

or

$$v_C(t) = \frac{v_{+\text{rail}}R_\text{b} + v_\text{ctrl}R_\text{a}}{R_\text{a} + R_\text{b}} = \left[\frac{v_{-\text{rail}}R_\text{b} + v_\text{ctrl}R_\text{a}}{R_\text{a} + R_\text{b}} - v_{+\text{rail}}\right]e^{-t/\tau} + v_{+\text{rail}}$$

or

$$\ln\left(\frac{\dfrac{v_{+\text{rail}}R_\text{b} + v_\text{ctrl}R_\text{a}}{R_\text{a} + R_\text{b}} - v_{+\text{rail}}}{\dfrac{v_{-\text{rail}}R_\text{b} + v_\text{ctrl}R_\text{a}}{R_\text{a} + R_\text{b}} - v_{+\text{rail}}}\right) = -t/\tau$$

or

$$t = -\tau\ln\left(\frac{\dfrac{v_{+\text{rail}}R_\text{b} + v_\text{ctrl}R_\text{a}}{R_\text{a} + R_\text{b}} - v_{+\text{rail}}}{\dfrac{v_{-\text{rail}}R_\text{b} + v_\text{ctrl}R_\text{a}}{R_\text{a} + R_\text{b}} - v_{+\text{rail}}}\right) = \tau\ln\left(\frac{\dfrac{v_{-\text{rail}}R_\text{b} + v_\text{ctrl}R_\text{a}}{R_\text{a} + R_\text{b}} - v_{+\text{rail}}}{\dfrac{v_{+\text{rail}}R_\text{b} + v_\text{ctrl}R_\text{a}}{R_\text{a} + R_\text{b}} - v_{+\text{rail}}}\right)$$

or

$$t = \tau\ln\left(\frac{v_{-\text{rail}}R_\text{b} + v_\text{ctrl}R_\text{a} - v_{+\text{rail}}(R_\text{a} + R_\text{b})}{v_{+\text{rail}}R_\text{b} + v_\text{ctrl}R_\text{a} - v_{+\text{rail}}(R_\text{a} + R_\text{b})}\right)$$

or

$$t = \tau\ln\left(\frac{v_{-\text{rail}}\dfrac{R_\text{b}}{R_\text{a}} + v_\text{ctrl} - v_{+\text{rail}}(1 + \dfrac{R_\text{b}}{R_\text{a}})}{v_\text{ctrl} - v_{+\text{rail}}}\right) = \tau\ln\left(\frac{v_\text{ctrl} - v_{+\text{rail}} - v_{+\text{rail}}\dfrac{R_\text{b}}{R_\text{a}} + v_{-\text{rail}}\dfrac{R_\text{b}}{R_\text{a}}}{v_\text{ctrl} - v_{+\text{rail}}}\right)$$

or, reversing signs in the numerator and denominator,

$$t = \tau\ln\left(\frac{v_{+\text{rail}} - v_\text{ctrl} + (v_{+\text{rail}} - v_{-\text{rail}})\dfrac{R_\text{b}}{R_\text{a}}}{v_{+\text{rail}} - v_\text{ctrl}}\right) = \tau\ln\left(1 + \frac{v_{+\text{rail}} - v_{-\text{rail}}}{v_{+\text{rail}} - v_\text{ctrl}} \cdot \frac{R_\text{b}}{R_\text{a}}\right).$$

If we set $v_\text{ctrl} = 0\text{V}$, we obtain the following simplified form.

$$t = \tau\ln\left(1 + \frac{v_{+\text{rail}} - v_{-\text{rail}}}{v_{+\text{rail}}} \cdot \frac{R_\text{b}}{R_\text{a}}\right) = \tau\ln\left(1 + \left[1 - \frac{v_{-\text{rail}}}{v_{+\text{rail}}}\right] \cdot \frac{R_\text{b}}{R_\text{a}}\right)$$

For $R_a = R_b$, as in Figs. 2 and 3,

$$t = \tau \ln\left(2 - \frac{v_{-\text{rail}}}{v_{+\text{rail}}}\right).$$

For the waveforms in Fig. 2, we have the following ratio of rail voltages:

$$\frac{v_{-\text{rail}}}{v_{+\text{rail}}} = \frac{-5\,\text{V}}{3\text{V}} = -\frac{5}{3}.$$

For the waveforms in Fig. 2, we have the following calculation:

$$t = \tau \ln\left(2 - -\frac{5}{3}\right) = \tau \ln\left(\frac{11}{3}\right) \approx \tau \cdot 1.3 \text{ for output high,}$$

and

$$t = \tau \ln(2 - -0.6) = \tau \ln(2.6) \approx \tau \cdot 0.95 \text{ for output low.}$$

REF: [1] https://www.fairchildsemi.com/datasheets/1N/1N914.pdf (accessed 23 July 2017)