CIRCUIT: The Wien bridge oscillator, see Fig. 1, consists of two voltage dividers. It oscillates (approximately) sinusoidally at the frequency that produces the same voltage out of both voltage dividers. When the two waveforms from the voltage divider are the same, the op-amp sees two waveforms that differ by the offset voltage of the op-amp, which is nonzero. This causes the output waveform at that frequency to be large, since changing the size of the output signal cannot eliminate the offset. This renders the negative feedback ineffective, and the open-loop gain of the op-amp applies to the offset voltage.

The Wien-bridge oscillator circuit is followed by a voltage follower with a DC-blocking capacitor.

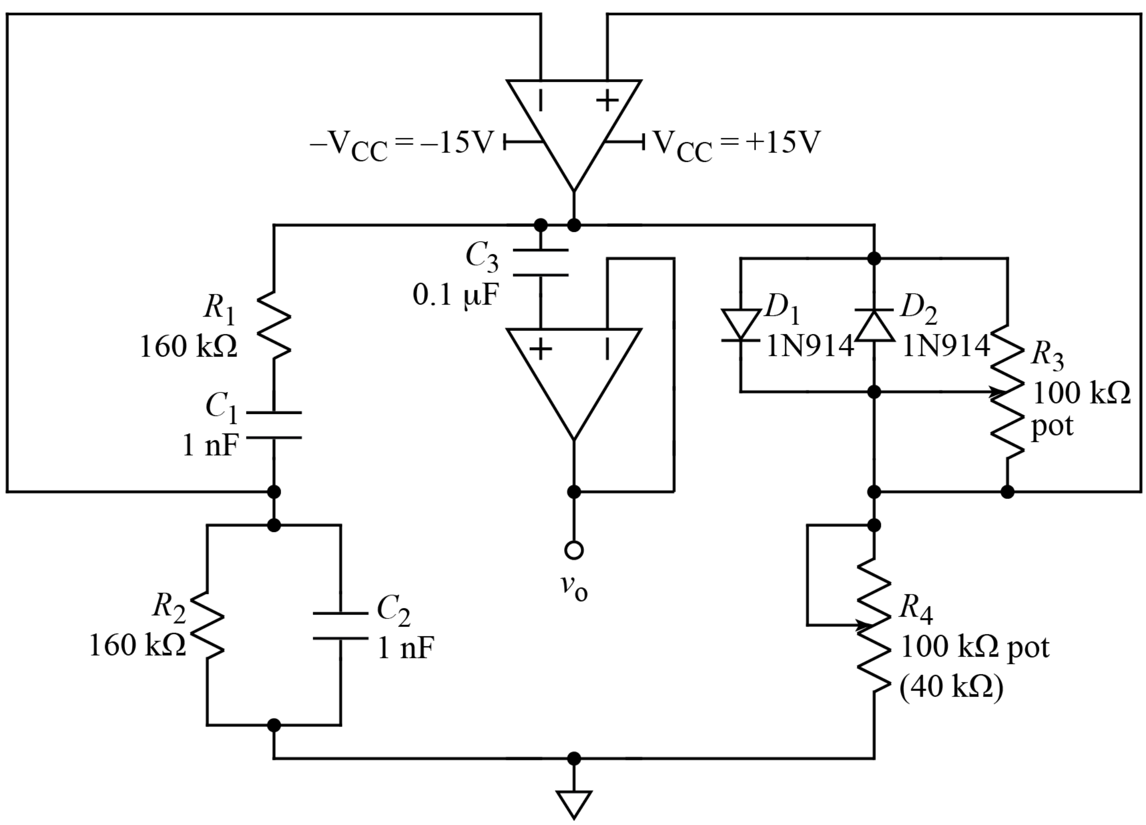

Fig. 1. Wien-bridge oscillator.

The diodes keep the output waveform from getting too large.

In terms of poles, the circuit has poles just to the right of the real axis. The diodes ensure that the poles are just to the right of the real axis at all times, preventing the output waveform from dying out if the poles were to drift into the left half-plane.

Analysis of circuit (skip to "**To adjust the oscillator**" if desired):

We equate the voltages out of the two voltage dividers and perform many steps of algebra to find the poles.

$$\frac{R_4}{R_3 + R_4} = \frac{R_2 \,||\, \frac{1}{sC_2}}{R_1 + \frac{1}{sC_1} + R_2 \,||\, \frac{1}{sC_2}}$$

or

$$\frac{1}{1 + \frac{R_3}{R_4}} = \frac{1}{1 + \frac{R_1 + \frac{1}{sC_1}}{R_2 \,||\, \frac{1}{sC_2}}}$$

or

$$1 + \frac{R_1 + \frac{1}{sC_1}}{R_2 \,||\, \frac{1}{sC_2}} = 1 + \frac{R_3}{R_4}$$

or

$$\frac{R_1 + \frac{1}{sC_1}}{R_2 \,||\, \frac{1}{sC_2}} = \frac{R_3}{R_4}$$

or

$$R_1 + \frac{1}{sC_1} = \frac{R_3}{R_4} \cdot R_2 \,||\, \frac{1}{sC_2}$$

or

$$R_1 + \frac{1}{sC_1} = \frac{R_3}{R_4} \cdot R_2 \,||\, \frac{1}{sC_2}$$

or

$$R_1 + \frac{1}{sC_1} = \frac{R_3}{R_4} \cdot \frac{\dfrac{R_2}{sC_2}}{R_2 + \dfrac{1}{sC_2}} = \frac{R_3}{R_4} \cdot \frac{R_2}{1 + sR_2C_2}$$

or

$$\frac{1 + sR_1C_1}{sC_1} = \frac{R_3}{R_4} \cdot \frac{R_2}{1 + sR_2C_2}$$

or

$$s^2 R_1C_1R_2C_2 + s(R_1C_1 + R_2C_2) + 1 = sR_2C_1\frac{R_3}{R_4}$$

or

$$s^2 + s\left(\frac{1}{R_2C_2} + \frac{1}{R_1C_1}\right) + \frac{1}{R_1C_1R_2C_2} = s\frac{1}{R_1C_2}\frac{R_3}{R_4}$$

or

$$s^2 + s\left(\frac{1}{R_2C_2} + \frac{1}{R_1C_1} - \frac{1}{R_1C_2}\frac{R_3}{R_4}\right) + \frac{1}{R_1C_1R_2C_2} = 0$$

or

$$s^2 + s\left(\frac{1}{\tau_{22}} + \frac{1}{\tau_{11}} - \frac{1}{\tau_{12}}\frac{R_3}{R_4}\right) + \frac{1}{\tau_{11}\tau_{22}} = 0$$

or

$$s = -\frac{1}{2}\left(\frac{1}{\tau_{11}} + \frac{1}{\tau_{22}} - \frac{1}{\tau_{12}}\frac{R_3}{R_4}\right) \pm \sqrt{\left[\frac{1}{2}\left(\frac{1}{\tau_{11}} + \frac{1}{\tau_{22}} - \frac{1}{\tau_{12}}\frac{R_3}{R_4}\right)\right]^2 - \frac{1}{\tau_{11}\tau_{22}}}$$

For purely imaginary roots, we want the real part (in front of the square root), to be zero. In actuality, we want the real part to be just slightly positive so our poles will be in the right half-plane.

$$-\frac{1}{2}\left(\frac{1}{\tau_{11}}+\frac{1}{\tau_{22}}-\frac{1}{\tau_{12}}\frac{R_3}{R_4}\right)\cong 0$$

or

$$\left(\frac{1}{\tau_{11}}+\frac{1}{\tau_{22}}-\frac{1}{\tau_{12}}\frac{R_3}{R_4}\right)=0$$

or

$$\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}-\frac{1}{R_1C_2}\frac{R_3}{R_4}\right)=0$$

or

$$R_1C_1+R_2C_2-R_2C_1\frac{R_3}{R_4}=0$$

or

$$\frac{R_1}{R_2}+\frac{C_2}{C_1}-\frac{R_3}{R_4}=0$$

or

$$\frac{R_3}{R_4}=\frac{R_1}{R_2}+\frac{C_2}{C_1}$$

For the roots to be in the right half-plane, R_3/R_4 must be larger than the right side of the equation. The diodes in the circuit, move the poles toward the left half-plane by lowering the value of R_3 when the amplitude of the output signal gets too large.

$$s=\pm j\sqrt{\frac{1}{\tau_{11}\tau_{22}}}=\pm j\sqrt{\frac{1}{R_1C_1R_2C_2}}$$

If $C_1 = C_2 = 1$ nF, we get poles at audio frequencies using R's that draw only modest current from the op-amp.

$$s=\pm j\frac{1}{C_1}\sqrt{\frac{1}{R_1R_2}}=\pm j1\,\text{Gr/s}\sqrt{\frac{1}{R_1R_2}}$$

For example, if we want an oscillation frequency at 1 kHz, we have the following calculations:

$$s = \pm j1\,\mathrm{Gr/s}\sqrt{\frac{1}{R_1R_2}} = \pm j1\,\mathrm{Gr/s}\sqrt{\frac{1}{R_1R_2}} = \pm j1\,\mathrm{k\,Hz}\cdot 2\pi$$

or

$$\sqrt{\frac{1}{R_1R_2}} = \frac{1\,\mathrm{k\,Hz}\cdot 2\pi}{1\,\mathrm{Gr/s}}$$

or

$$R_1R_2 = \left(\frac{1\,\mathrm{Gr/s}}{1\,\mathrm{k\,Hz}\cdot 2\pi}\right)^2 \cong (160\,\mathrm{k}\Omega)^2$$

For $R_1 = R_2$ we have a solution with a standard 5% resistor value:

$R_1 = R_2 = 160\ \mathrm{k}\Omega$

For this solution, we need $R_3 = 2R_4$ for poles on the imaginary axis.

Now we turn to the question of how to pick R_3 so that the peak current through R_3 will result in a voltage across the diodes that will conduct enough current to move poles to the left half-plane. That is, the current in the diodes should be a significant proportion of the current in R_3.

We assume a desired magnitude 1 V sinusoidal output.

TABLE I: 1N914 DIODE *I-V* DATA (FAIRCHILD) [1]

i_d	v_d
1 μA	0.273 V
5 μA	0.352 V
10 μA	0.380 V
100 μA	0.505 V

Since $R_3 = 2R_4$ and the output voltage is across R_4, we will have 1 V across R_4 and 2 V across R_3. Say the value of $R_3 = 100$ kΩ.

$$i_{R3} = \frac{2\,\text{V}}{100\,\text{k}\Omega} = 20\,\mu\text{A}$$

Say we wish to have 25% of the current in R_3, i.e., 5 μA, flowing in the diodes. This effectively reduces the resistance to an equivalent of $R_3 = 80$ kΩ. Thus, we adjust R_4 to 40 kΩ so it will be half the value of the equivalent R_3. From the data for the 1N914 diode, the voltage drop across the diodes will be 0.352 V.

The part of R_3 that drops 0.352 V when the current is 10 μA is given by Ohm's law:

$$R_{3\ \text{top}} = \frac{0.352\,\text{V}}{10\,\mu\text{A}} = 35.2\,\text{k}\Omega$$

To adjust the oscillator, first adjust R_4 until the circuit oscillates. Then adjust R_3 until the amplitude of the oscillation is as desired.

REF: [1] https://www.fairchildsemi.com/datasheets/1N/1N914.pdf (accessed 23 July 2017)