Load Flow Analysis
Computation Techniques

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Example 10.2

In Example 10.1 we found an explicit solution for the power flow equations. Now to illustrate the technique we would like to use Gauss iteration to solve the same problem. Suppose that $P_{01} = 0.5$. We show this in Figure E10.2. The problem can be stated as follows: given $V_1 = 1/\Omega^\circ$ and $S_2 - S_{02} = -0.5$, find $S_1$ and $V_2$, using Gauss iteration.

**Solution** We iterate on $V_2$ using (10.8). With $n = 2$ there is only one equation.

Figure E10.2

$V_2^{*+1} = \frac{1}{Y_{22}} \left[ \frac{S_2^*}{(V_2^{*})^2} - Y_{21} V_1 \right]

We next calculate the elements of $Y_{bus}$. $Z_L = j0.5$ implies that

$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$

Thus, putting in the values for $S_2$, $V_1$, $Y_{11}$, and $Y_{12}$, we get

$V_2^{*+1} = \frac{-0.25}{j(1/\Omega^\circ)^2} + 1.0$

Starting with a guess, $V_2^{0} = 1/\Omega^\circ$, we get convergence in about six steps. Even with a very poor guess we get convergence in about eight steps. The results are listed in Table E10.2. The exact solution from Example 10.1 is

$V_2 = 0.965976 \angle -15.000000^\circ$

**Note:** As we showed in Example 10.1, there are two solutions to the equation. The second is $V_2 = 0.258819 \angle -75.000000^\circ$. If we try to converge to this solution by...
Gauss iteration, we fail. Even with an initial choice as close as $V_2^0 = 0.25 \angle -75^\circ$, we reach the larger of the two solutions. In table E10.2 we show the iterations for two different starting points.

<table>
<thead>
<tr>
<th>Iteration number</th>
<th>$V_2$</th>
<th>$V_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 $\angle 0^\circ$</td>
<td>0.1 $\angle 0^\circ$</td>
</tr>
<tr>
<td>1</td>
<td>1.030776 $\angle -14.036249$</td>
<td>2.692582 $\angle -68.198591$</td>
</tr>
<tr>
<td>2</td>
<td>0.970143 $\angle -14.036249$</td>
<td>0.914443 $\angle -2.161079$</td>
</tr>
<tr>
<td>3</td>
<td>0.970261 $\angle -14.931409$</td>
<td>1.026705 $\angle -15.431731$</td>
</tr>
<tr>
<td>4</td>
<td>0.966235 $\angle -14.995078$</td>
<td>0.964213 $\angle -14.089103$</td>
</tr>
<tr>
<td>5</td>
<td>0.966236 $\angle -14.995078$</td>
<td>0.970048 $\angle -15.025221$</td>
</tr>
<tr>
<td>6</td>
<td>0.965948 $\angle -14.995072$</td>
<td>0.965813 $\angle -14.934752$</td>
</tr>
<tr>
<td>7</td>
<td>0.966221 $\angle -15.001783$</td>
<td>0.965918 $\angle -14.995310$</td>
</tr>
<tr>
<td>8</td>
<td>0.965918 $\angle -14.995310$</td>
<td>0.966221 $\angle -15.001783$</td>
</tr>
</tbody>
</table>
A Different Case: Case II

Case I:
So far, the analysis we have done is good for a system with a slack bus and the remaining buses are PQ bus.
Given quantities: $V_1, S_2(P_2, Q_2), S_3(P_3, Q_3), \ldots, S_n(P_n, Q_n)$
We need to find $S_1, V_2, V_3, \ldots, V_n$

Case II:
Now we have some PV buses as well
Given quantities: $V_1, (P_2, |V_2|), \ldots, (P_m, |V_m|), S_{m+1}, \ldots, S_n$
We need to find $S_1, (Q_2, \theta_2), \ldots, (Q_m, \theta_m), V_{m+1}, \ldots, V_n$
Computational Steps in Case II

\[ V_{i}^{(k+1)} = \frac{P_i - jQ_i}{V_i^{*(k+1)}} - \sum_{j=1, j\neq i}^{n} Y_{ij} V_j^{(k+1)} \frac{Y_{ii}}{} \]

\[ Q_i^{(k)} = -\text{Im} \left[ V_i^{*(k)} \sum_{j=1}^{N} (Y_{ij} \cdot V_j^{(k)}) \right] \]

1. Assume an angle for V as \(|V|\) is known
2. Use the V to determine Q and find a new V
3. Take the angle from the new V, and use the known value of V.
4. Repeat step 2, 3 until the error is within the margin.
Example 10.3

We consider a simple example illustrating Gauss iteration for Case II (see E10.3). The problem can be stated as follows given $V_i = 1$ and $P_i = -0.75, W_j$ and $S_i, Q_i, \theta_i$, using Gauss iteration.

\[ \begin{align*}
V_1 &= 10^2 \\
S_1 &= 0.25 + i Q_{02} \\
S_2 &= 0.25 + i Q_{02} \\
S_1 &= V_i [Y_{11} V_i^* + Y_{12} V_2^*] \\
&= j2 - j2 / 22.0238^* \\
&= 0.7641 / 11.0119^* = 0.7500 + j0.1459
\end{align*} \]

which conforms to the expected result.

**Figure E10.3**

Solution: The iteration formula (10.14) with $Y_i = -j2, Y_0 = j2$ gives

\[ \begin{align*}
\bar{V}^{t+1}_i &= \frac{1}{j2} \left[ \begin{array}{c}
-0.75 - /Q_2 \\
(V_i)^* - j2
\end{array} \right] \\
&= 1 + 0.75 + jQ_2 \\
&= j2(V_i)^*
\end{align*} \]

where we estimate $Q_2$ using (10.15), that is,

\[ Q_2 = \text{Im} \left[ V_i (Y_{11} V_i^* + Y_{12} (V_2)^*) \right] = \text{Im} (-j2V_2^* + j2 (V_2)^*) = 2(1 - \text{Re} V_i^*) \]

Starting with a guess, $V_2^* = 1 / \theta^*$, we obtain the iterations shown in Table E10.3. Convergence is obtained after four iterations. The exact solution may easily be found without iteration in this simple case using (4.54) and (4.55). The result is

\[ \theta_i = \bar{V}_1 = -22.0238^* \quad Q_i = Q_{01} = 0.1460 \]

**TABLE E10.3**

<table>
<thead>
<tr>
<th>Iteration number</th>
<th>$V_1^*$</th>
<th>$Q_i$</th>
<th>$\bar{V}_2^{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1.0680 / -20.5560°</td>
</tr>
<tr>
<td>1</td>
<td>0.1273</td>
<td>1.0000 / -21.5522°</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.1446</td>
<td>1.0000 / 22.0238°</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.1459</td>
<td>1.0000 / 22.0238°</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.1459</td>
<td>1.0000 / 22.0238°</td>
<td></td>
</tr>
</tbody>
</table>

To complete the problem, we can solve for $S_i$ using (10.3):