Three Phase Circuit Parameters (Cont.)

Prof. Faisal Khan
ECE Department, University of Utah
Delta to Wye Transformation

A delta load can be transformed into a wye load. It provides the flexibility to analyze a three phase circuit in single phase configuration.
Delta to Wye Transformation

The Two Above Sources are Equivalent in Balanced Systems
(i.e., same line currents $I_a$, $I_b$, $I_c$ and phase-to-phase voltages $V_{ab}$, $V_{bc}$, $V_{ca}$ in both cases)
A balanced 3φ system can be modeled as a 1φ system.

• In wye connection
  ▪ Line-to-line voltage (source) = $\sqrt{3} \times$ Line-to-neutral (phase) voltage (load)
  ▪ Line current (source) = Phase current (load)

• In delta connection
  ▪ Line current (source) = $\sqrt{3} \times$ Phase current (load)
  ▪ Line-to-line voltage (source) = Phase voltage (load)

Wye connection and delta connection:
Three Phase Parameters

First, some trigonometry identities:

\[
\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)
\]

\[
\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)
\]

\[
\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)
\]

\[
\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)
\]

Because they form a balanced set, the a-b-c currents sum to zero. Thus, there is no return current through the neutral or ground, which reduces wiring losses.

\[
i_a(t) = I \cos(\omega t + \theta), \quad i_b(t) = I \cos(\omega t + \theta - 120^\circ), \quad i_c(t) = I \cos(\omega t + \theta + 120^\circ),
\]

\[
i_N(t) = i_a(t) + i_b(t) + i_c(t) = I \left[ \cos(\omega t + \theta) + \cos(\omega t + \theta - 120^\circ) + \cos(\omega t + \theta + 120^\circ) \right].
\]
Neutral Current in a Balanced Three Phase System

\[ i_N(t) = \frac{I}{2} \left[ \cos(\omega t) \cos(\theta) - \sin(\omega t) \sin(\theta) ight. \\
+ \cos(\omega t) \cos(\theta - 120^\circ) - \sin(\omega t) \sin(\theta - 120^\circ) \\
+ \cos(\omega t) \cos(\theta + 120^\circ) - \sin(\omega t) \sin(\theta + 120^\circ) \right], \]

\[ i_N(t) = \frac{I}{2} \left[ \cos(\omega t) \cos(\theta) - \sin(\omega t) \sin(\theta) ight. \\
+ \cos(\omega t) \left[ \cos(\theta) \cos(120^\circ) + \sin(\theta) \sin(120^\circ) \right] - \sin(\omega t) \left[ \sin(\theta) \cos(120^\circ) - \cos(\theta) \sin(120^\circ) \right] \\
+ \cos(\omega t) \left[ \cos(\theta) \cos(120^\circ) - \sin(\theta) \sin(120^\circ) \right] - \sin(\omega t) \left[ \sin(\theta) \cos(120^\circ) + \cos(\theta) \sin(120^\circ) \right] \right], \]

\[ i_N(t) = \frac{I}{2} \cos(\omega t) \left[ \cos(\theta) \left[ 1 + \cos(120^\circ) + \cos(120^\circ) \right] + \sin(\theta) \left[ \sin(120^\circ) - \sin(120^\circ) \right] \right] \\
- \frac{I}{2} \sin(\omega t) \left[ \sin(\theta) \left[ 1 + \cos(120^\circ) + \cos(120^\circ) \right] - \cos(\theta) \left[ \sin(120^\circ) - \sin(120^\circ) \right] \right], \]

\[ i_N(t) = \frac{I}{2} \cos(\omega t) \left\{ \cos(\theta) \left[ 1 - \frac{1}{2} - \frac{1}{2} \right] + \sin(\theta) \left[ \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right] \right\} \\
- \frac{I}{2} \sin(\omega t) \left\{ \sin(\theta) \left[ 1 - \frac{1}{2} - \frac{1}{2} \right] - \cos(\theta) \left[ \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right] \right\}. \]

Thus \( i_N(t) = 0 \).
More trigonometry identities:

\[
\cos(A) \cos(B) = \frac{1}{2} \left[ \cos(A - B) + \cos(A + B) \right]
\]

\[
\sin(A) \sin(B) = \frac{1}{2} \left[ \cos(A - B) - \cos(A + B) \right]
\]

The instantaneous power is constant

\[
v_a(t) = V \cos(\omega t + \delta), \quad v_b(t) = V \cos(\omega t + \delta - 120^\circ), \quad v_c(t) = V \cos(\omega t + \delta + 120^\circ),
\]

\[
p_{tot}(t) = p_a(t) + p_b(t) + p_c(t),
\]

\[
= VI \left[ \cos(\omega t + \delta) \cos(\omega t + \theta) + \cos(\omega t + \delta - 120^\circ) \cos(\omega t + \theta - 120^\circ) \\
+ \cos(\omega t + \delta + 120^\circ) \cos(\omega t + \theta + 120^\circ) \right],
\]
Three Phase Power (Cont.)

\[
\frac{VI}{2} \left[ \cos(\delta - \theta) + \cos(2\omega t + \delta + \theta) + \cos(\delta - \theta) + \cos(2\omega t + \delta + \theta - 240^\circ) \right. \\
+ \cos(\delta - \theta) + \cos(2\omega t + \delta + \theta + 240^\circ) \right], \\
= \frac{3VI}{2} \cos(\delta - \theta) + \frac{VI}{2} \left[ \cos(2\omega t + \delta + \theta) + \cos(2\omega t + \delta + \theta + 120^\circ) + \cos(2\omega t + \delta + \theta - 120^\circ) \right].
\]

The terms in side the bracket form a balanced set, and thus equal zero. Thus, \( p_{tot}(t) \) is a constant and equals its average value, which is

\[
p_{tot}(t) = p_a(t) + p_b(t) + p_c(t) = \frac{3VI}{2} \cos(\delta - \theta) = 3V_{rms} I_{rms} \cos(\delta - \theta).
\]
4.17A on the 240V side corresponds to 0.14A on the 7.2kV side.

**Figure 2.11** Three-wire, single-phase power drop, including the wiring in the breaker box to feed 120-V and 240-V circuits in the house.

**Figure 2.12** Waveforms for ±120 V and the difference between them creating 240 V.
Most commercial buildings, like MEB, are served by a 12.47kV (line-to-line) to 480V (line-to-line) three-phase transformer, delta-connected on the 12.47kV side, and grounded-wye connected on the 480V side.

1kVA to the customer corresponds to 1.20A in each of the three phases.

Most lighting in commercial buildings is 277V fluorescent – that’s the voltage behind the switch plate cover. 120V outlets are served by 480:120V step down transformers on each floor of the building, often located in utility closets in two diagonally-opposite corners of each floor.