### 7.6.3 Decibel Scale for Power Ratios

The unit for power $P$ is watts (W). In many engineering problems, the quantity of interest is the ratio of two power levels, $P_1$ and $P_2$, such as the incident and reflected powers on a transmission line, and often the ratio $P_1 / P_2$ may vary over several orders of magnitude. The decibel (dB) scale is logarithmic, thereby providing a convenient representation of the power ratio, particularly when numerical values of $P_1 / P_2$ are plotted against some variable of interest. If

$$G = \frac{P_1}{P_2},$$  \hspace{1cm} (7.110)

then

$$G \text{ [dB]} = 10 \log G = 10 \log \left( \frac{P_1}{P_2} \right) \text{ (dB).}$$  \hspace{1cm} (7.111)

Table 7-2 provides a comparison between some values of $G$ and the corresponding values of $G$ [dB]. Even though decibels are defined for power ratios, they can sometimes be used to represent other quantities. For example, if $P_1 = V_1^2 / R$ is the power dissipated in a resistor $R$ with voltage $V_1$ across it at time $t_1$, and $P_2 = V_2^2 / R$ is the power dissipated in the same resistor at time $t_2$, then

$$G \text{ [dB]} = 10 \log \left( \frac{P_1}{P_2} \right) = 10 \log \left( \frac{V_1^2}{V_2^2} \right) = 20 \log \left( \frac{V_1}{V_2} \right) = 20 \log(g) \triangleq g \text{ [dB]},$$  \hspace{1cm} (7.112)

where $g = V_1 / V_2$ is the voltage ratio. Note that for voltage (or current) ratios the scale factor is 20 rather than 10, which results in $G \text{ [dB]} = g \text{ [dB]}$.

<table>
<thead>
<tr>
<th>$G$</th>
<th>$G \text{ [dB]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^4</td>
<td>10 × dB</td>
</tr>
<tr>
<td>4</td>
<td>6 dB</td>
</tr>
<tr>
<td>2</td>
<td>3 dB</td>
</tr>
<tr>
<td>1</td>
<td>0 dB</td>
</tr>
<tr>
<td>0.5</td>
<td>-3 dB</td>
</tr>
<tr>
<td>0.25</td>
<td>-6 dB</td>
</tr>
<tr>
<td>0.1</td>
<td>-10 dB</td>
</tr>
<tr>
<td>10^{-3}</td>
<td>-30 dB</td>
</tr>
</tbody>
</table>

The attenuation rate, representing the rate of decrease of the magnitude of $S_{aw}(z)$ as a function of propagation distance, is defined as

$$A = 10 \log \left( \frac{S_{aw}(z)}{S_{aw}(0)} \right) = 10 \log(e^{-2\alpha z}) = -20\alpha z \text{ log } e = -8.68\alpha z = -\alpha \text{ [dB/m]} z \text{ (dB).}$$  \hspace{1cm} (7.113)

where

$$\alpha \text{ [dB/m]} \triangleq 8.68\alpha \text{ [Np/m].}$$  \hspace{1cm} (7.114)

We also note that, since $S_{aw}(z)$ is directly proportional to $|E(z)|^2$,

$$A = 10 \log \left( \frac{|E(z)|^2}{|E(0)|^2} \right) = 20 \log \left( \frac{|E(z)|}{|E(0)|} \right) \text{ (dB).}$$  \hspace{1cm} (7.115)

### Example 7-6 Power Received by a Submarine Antenna

A submarine at a depth of 200 m uses a wire antenna to receive signal transmissions at 1 kHz. Determine the power density incident upon the submarine antenna due to the EM wave of Example 7-4.
GLOSSARY OF IMPORTANT TERMS

Solution: From Example 7-4, \( |E_0| = |E_{x0}| = 4.44 \) (mV/m), \( \alpha = 0.126 \) (Np/m), and \( \eta_c = 0.044 \sqrt{25\pi} \) (\( \Omega \)). Application of Eq. (7.109) gives

\[
S_{av}(z) = \frac{z}{2|\eta_c|} \frac{|E_0|^2}{e^{-2az} \cos \theta_y} \\
= \frac{z}{2} \left( \frac{4.44 \times 10^{-3}}{2 \times 0.044} \right)^2 e^{-0.252z} \cos 45^\circ \\
= \frac{z}{16\omega} e^{-0.252z} \quad (\text{mW/m}^2)
\]

At \( z = 200 \) m, the incident power density is

\[
S_{av} = \frac{z}{16\omega} (0.16 \times 10^{-3} e^{-0.252 \times 200}) \\
= 2.1 \times 10^{-26} \quad (\text{W/m}^2).
\]

EXERCISE 7.10 Convert the following values of the power ratio \( G \) from natural numbers to decibels: (a) 2.3, (b) \( 4 \times 10^3 \), (c) \( 3 \times 10^{-2} \).

Ans. (a) 3.0 dB, (b) 36 dB, (c) -13.2 dB.

EXERCISE 7.11 Find the voltage ratio \( g \) in natural units corresponding to the following decibel values of the power ratio \( G \): (a) 23 dB, (b) -14 dB, (c) -3.6 dB.

Ans. (a) 14.13, (b) 0.2, (c) 0.66.

CHAPTER HIGHLIGHTS

- Wave polarization describes the shape and locus of the tip of the \( \mathbf{E} \) vector at a given point in space as a function of time. The polarization state, which may be linear, circular, or elliptical, is governed by the ratio of the magnitudes of and the difference in phase between the two orthogonal components of the electric field vector.

- Media are classified as lossless, low-loss, quasi-conducting, or good conducting on the basis of the ratio \( \varepsilon''/\varepsilon' = \sigma/\omega \).

- Unlike the d-c case, wherein the current flowing through a wire is distributed uniformly across its cross section, in the a-c case most of the current is concentrated along the outer perimeter of the wire.

- Power density carried by a plane EM wave traveling in an unbounded medium is akin to the power carried by the voltage/current wave on a transmission line.

GLOSSARY OF IMPORTANT TERMS

Provide definitions or explain the meaning of the following terms:

guided medium
unbounded medium
spherical wave
uniform plane wave
complex permittivity \( \varepsilon_c \)
wave number \( k \)
TEM wave
intrinsic impedance \( \eta \)
wave polarization
elliptical polarization
circular polarization
linear polarization
LHC and RHC polarizations
attenuation constant \( \alpha \)
phase constant \( \beta \)
skin depth \( \delta \)
low-loss dielectric
quasi-conductor
good conductor