ECE 5570 - Lab 4
Closed-Loop Control of a Stepper Motor

Objectives

The objective of this lab is to develop and test a closed-loop control algorithm for a stepper motor. First, field weakening is combined with a DQ transformation to maximize the achievable speed and to guarantee synchronism of the motor with the rotating magnetic field. Then, a PID control law is implemented for position control.

1 Introduction

The DQ transformation is given by

\[ v_d = \cos(n_p \theta)v_A + \sin(n_p \theta)v_B \]
\[ v_q = -\sin(n_p \theta)v_A + \cos(n_p \theta)v_B \]

with inverse

\[ v_A = \cos(n_p \theta)v_d - \sin(n_p \theta)v_q \]
\[ v_B = \sin(n_p \theta)v_d + \cos(n_p \theta)v_q \]

In the DQ coordinates, the stepper motor model is

\[ L \frac{di_d}{dt} = v_d - R_i d + n_p \omega L_i q \]
\[ L \frac{di_q}{dt} = v_q - R_i q - n_p \omega L_i d - K\omega \]
\[ J \frac{d\omega}{dt} = K_i q - \tau_{LF} \]
\[ \frac{d\theta}{dt} = \omega \]

where \( v_d \) and \( v_q \) (V) are the DQ voltages, \( i_d \) and \( i_q \) (A) are the DQ currents, \( \omega \) (rad/s) is the angular velocity of the motor, and \( \theta \) (rad) is the angular position.

The parameters are:

R (\Omega) the resistance of each of the phase windings,
L (H) the inductance of each of the phase windings,
K (Nm/A or Vs) the torque/back-emf constant,
J \ (kg \ m^2) \ the \ rotor \ inertia, \\
\tau_{\text{fr}} \ (Nm) \ the \ load/ \ friction \ torque, \\
n_p \ the \ number \ of \ pole \ pairs

In steady-state operation, one has that

$$
\begin{pmatrix}
  i_d \\
  i_q
\end{pmatrix}
= \frac{1}{R^2 + (n_p \omega L)^2}
\begin{pmatrix}
  R & n_p \omega L \\
  -n_p \omega L & R
\end{pmatrix}
\begin{pmatrix}
  v_d \\
  v_q - K \omega
\end{pmatrix}
$$

At low speeds, it is reasonable to set \( v_d = 0 \), resulting in \( i_q = v_q / R \). At high speeds, however, the current \( i_q \) rapidly decreases for a given voltage because of inductive effects and because of the back-emf \( (K \omega) \). The speed at which inductive effects become significant is approximately a decade below \( \omega = R / (n_p L) \) and is usually low for stepper motors, because of their large inductance and number of steps. In order to increase the torque available, the current \( i_q \) may be maximized for a bounded amplifier voltage by setting a current \( i_d \) equal to

$$
i_d = -\frac{(n_p \omega L)(K \omega)}{R^2 + (n_p \omega L)^2}
$$

or a relationship between the voltages

$$
v_d = -\frac{n_p \omega L}{R} v_q
$$

2 Experiments

Equipment needed:

- Stepper motor,
- Brush DC motor,
- Encoder in a bracket,
- Dual power amplifier,
- Cable rack
- dSPACE kit which includes an encoder cable and I/O breakout box.
- You will also need a metal frame to mount the motors on, and a box with screws and a screwdriver.

2.1 Preliminary testing

Following the usual steps, test the operation of the Dspace D/A and encoder. Attach the stepper motor to the DC motor through a shaft coupler and connect the two phases of the stepper motor to the amplifiers. Test the stepper motor using experiment ECE5570_lab3.
2.2 Field Weakening

Modify the code for ECE5570_lab4.c to implement field weakening. Given a keyboard entry $v$, the voltages $v_d$ and $v_q$ should be given by

$$v_d = \frac{-n_p\omega L}{\sqrt{R^2 + (n_p\omega L)^2}} v$$

$$v_q = \frac{R}{\sqrt{R^2 + (n_p\omega L)^2}} v$$

The speed may be reconstructed by the usual backwards differentiation formula. Filtering is not necessary. Once the program is debugged, perform the following experiments:

- Apply steps of voltages $v = 5V, 10V, 15V, 20V, \text{and} 25V$. Observe that the response of the system from $v$ to $\omega$ is approximately linear. Also observe that the maximum speed reached is much higher than without the field weakening. Compare the maximum speed reached to that obtained with open-loop stepping in Lab 3.
- Apply a step of voltage equal to 25V. Considering that the response of the motor is approximately that of a first-order system, estimate the gain and the time constant.

2.3 Closed-loop DQ control

Modify the program to implement the same PIDF control as you implemented for the DC motor in Lab 2, but this time using the variable $v$ as a control input. Use the previous procedure to tune the parameters: first look for a good value of $k_p$, then adjust $k_p$ and $k_i$ using the formulas resulting from the principle of the symmetric optimum. Set $k_r = 0.5$. Subsequent fine-tuning of the parameters may be useful but possibly not necessary. In your report, show responses to the following reference inputs:

- Step to 90 and then -90 degrees,
- Step to 3600 and then 0 degrees,
- Step to 1, 2, 3, 4, 5, 6, 7, and 8 degrees (the motor resolution is approx 7.5 degrees so these step are referred to as microsteps).
3 Report at a Glance

Be sure to include:

- Plot of the speed for multiple steps of voltage $v$.
- Plot of the speed for a large step of voltage $v$ and estimates of the gain and of the time constant.
- Value of the derivative gain and calculation of the PI parameters.
- Plots of the position for quarter-turn moves, 10-revolution moves, and micro-steps.
- Comments.