Objectives

The objective of the lab is to measure the torque curves of induction motors. Acceleration experiments are used to reconstruct approximately the torque curves, assuming that the load torque is constant and that a steady-state approximation is valid. The dependency of the torque on the electrical frequency is also investigated.

1. Introduction

The dynamic response of an induction motor is considerably more complex than the response of a brush DC motor or a permanent-magnet synchronous motor. A simple model includes at least five nonlinear differential equations. However, the torque curves obtained under steady-state assumptions provide valuable information for the design of slip control drives.

Consider a two-phase induction motor with sinusoidal phase voltages

\[
v_a = V_S \cos(\omega_e t),
\]

\[
v_b = V_S \sin(\omega_e t).
\]

letting \( V_S \) be the peak voltage and \( \omega_e \) be the electrical frequency. Assume that the electrical machine is in sinusoidal steady-state and that the speed is constant. Then, the electromagnetic torque is equal to

\[
\tau_e(\omega, \omega_e) = n_p \frac{M^2}{R_e} \frac{V_S^2}{\omega_e^2} \left( \frac{\omega_e - n_p \omega}{R_e - \sigma L_s \omega_e T_R \left( \omega_e - n_p \omega \right)} \right)^2 + \left( R_s T_R \left( \omega_e - n_p \omega \right) + L_s \omega_e \right)^2
\]

Although equation (2) is only valid under the steady-state assumptions, it is useful in other cases as well provided that the electrical and mechanical frequencies vary slowly.

Fig. 1 shows a set of torque curves for a typical induction motor. The torque is plotted as a function of the speed \( \omega \), expressed in rpm, for a given electrical frequency (the three curves correspond to electrical frequencies of 20 Hz, 40 Hz, and 60 Hz respectively). For the motor under consideration, the number of pole pairs is \( n_p = 1 \), and a frequency of 60 Hz corresponds to a synchronous speed \( \omega_s = n_p \omega_e \) equal to 3600 rpm. Note that the torque goes to zero when the speed reaches the synchronous speed. Close to the synchronous speed,
the torque is approximately linear in the difference between \( \omega_s \) and \( n_p \omega \), called the slip. Specifically, for small slip, the torque is approximately given by

\[
\tau_e(\omega, \omega_s) = n_p \frac{M^2}{R_R} V_s^2 \frac{\left(\omega_s - n_p \omega\right)}{\left(R_s\right)^2 + \left(L_s \omega_s\right)^2}
\]

Equation (3) as well as Fig. 1 show that the torque decreases rapidly when the electrical frequency increases.

2. Experiments

Equipment needed

- Induction motor,
- Encoder in a bracket,
- Dual power amplifier,
- Cable rack
- dSPACE kit which includes an encoder cable and I/O breakout box.
- You will also need a metal frame to mount the motors on, and a box with screws and a screwdriver.

2.1 Preliminary Testing

Carry out the usual testing procedure to check the encoder and power supply.
2.2 Torque Curves

The experiment ECE5570_lab5 applies two phase voltages of the form (1) to the motor, with \( V_S = 25 \) V. The operator enters a value for the electrical frequency expressed in terms of the synchronous speed in rpm. The induction motor in the lab has 1 pole pair \((n_p = 1)\), so that a frequency of 60 Hz corresponds to a synchronous speed of 3600 rpm. 60 Hz is also the nominal frequency for the motor under consideration.

**Important:** please note that the two windings of the induction motor are connected together. Although there are four banana plugs on the motor frame, the two middle plugs (colored blue) are connected to the same wire. Therefore, make sure that the grounds of both amplifiers (black outputs of the amplifiers) are connected to the blue plugs of the motor frame. Also, if the direction of rotation is negative in your initial experiments, swap the phases (preferably by swapping the DACH channels), so that the direction of rotation changes.

Using the experiment ECE5570_lab5, measure the response of the motor for sinusoidal voltages applied at \( t = 0 \). Determine the value of the speed in the steady-state (it is useful to average the velocity over a sufficient period of time after it has stabilized). Calculate the values of the slip and of the normalized slip in %.

Measuring the torque curves precisely would require a torque sensor. However, useful results can be obtained from the acceleration of the motor alone. The torque itself is not obtained. Rather, the torque divided by the inertia \( J \) is obtained, but that is all that is needed for control system design.

The acceleration of the motor can be reconstructed using numerical differentiation. It is necessary to filter the velocity, for example using a Butterworth filter of order 5 and bandwidth 25 Hz. One has that

\[
\frac{d \omega}{dt} = \frac{\tau_e (\omega, \omega_s)}{J} - \frac{\tau_{LF}}{J}
\]

assuming that the electromagnetic torque can be approximated by the steady-state value. Assuming that the load torque is constant (it consists mostly of Coulomb friction in the set-up), the torque, \( \tau_e (\omega, \omega_s) - \tau_{LF} \) divided by the inertia, can be deduced.

Plot \( d \omega / dt \) as a function of \( \omega - \omega \) (use the filtered velocity as well as acceleration for the plot). You should recognize the shape of the torque curves shown in the Fig. 1. For small slip and \( n_p = 1 \), one has that

\[
\frac{d \omega}{dt} = k \left( (\omega_e - \omega) - (\omega_e - \omega_{ss}) \right).
\]

where

\[
k = \frac{M^2}{J R_k} \left( \frac{V_s^2}{R_s} \right)^2 + \left( L_s \omega_s \right)^2
\]

and \( \tau_{LF} / J \) satisfies approximately

\[
\frac{\tau_{LF}}{J} = k (\omega_e - \omega_{ss}),
\]
with \( \omega_{ss} \) being the steady-state speed. From the plot of the acceleration, Find the value of the constant \( k \) such that the best fit is obtained for
\[
\frac{d\omega}{dt} = k \left( (\omega_e - \omega) - (\omega_e - \omega_{ss}) \right),
\] (8)
Perform this fit visually in Matlab. The estimate of \( k \) will be very approximate, but only an approximate number is needed for control design.

Repeat the estimation of the parameter \( k \) at 2400 rpm and 1200 rpm. Equation (3) predicts that the constant \( k \) is inversely proportional to \( 1 + (L_s/R_s)^2 \cdot (\omega_e)^2 \). From this fact, and from your measurements, determine a "ball park" estimate of the constant \( T_s = L_s / R_s \).

3. Report at a Glance

Be sure to include:

- Plot of speed vs. time for a synchronous speed of 3600 rpm and values of the steady-state speed, slip, and normalized slip.
- Plot of acceleration vs. slip for synchronous speeds of 3600 rpm, 2400 rpm, and 1200 rpm.
- Values of the steady-state slip and of the parameter \( k \).
- Calculation of the estimate of \( T_s = L_s / R_s \).
- Comments.