1. For the MOS differential pair with a common-mode voltage $V_{CM}$ applied, as shown below, let $V_{DD}=V_{SS}=1V$, $k_n'=0.4mA/V^2$, $(W/L)_{1,2}=12.5$, $V_t=0.5V$, $I_s=0.2mA$, and $R_D=10k\Omega$ (neglect channel-length modulation). Assume that the current source $I$ requires a minimum voltage of 0.4V to operate properly. (worth 2 problems)

(a) Find $V_{GS}$ for each transistor.
(b) For $V_{CM}=0$ find $V_s$, $I_{D1}$, $I_{D2}$, $V_{D1}$, and $V_{D2}$.
(c) Repeat (b) for $V_{CM}=0.3V$.
(d) Repeat (b) for $V_{CM}=-0.1V$.
(e) What is the highest permitted value of $V_{CM}$ for which Q1 and Q2 remain in saturation?
(f) If the current source requires a minimum of 0.2V across it to operate correctly, what is the lowest value allowed for $V_s$ and hence for $V_{CM}$?

\[ I_D = \frac{1}{2} \frac{k_n(W)}{L} V_{GS}^2 \]
\[ V_{ov} = \sqrt{\frac{2I_D}{k_n(W/L)}} = \sqrt{\frac{2(0.1mA)}{0.4mA/V^2(12.5)}} \]

\[ = 0.2V \]

\[ V_{D} = V_s + V_{ov} = 0.5 + 0.2 = 0.7V \]

(b) If $V_{CM} = 0$, 
- $V_{s1} = V_{s2} = V_G - V_{GS} = 0 - 0.7 = -0.7V$
- $I_{D1} = I_{D2} = \frac{0.2mA}{2} = 0.1mA$
- $V_{D1} = V_{D2} = V_{DD} - I_{D1}R_D$
  \[ = 1V - (0.1mA)(10K) = 0V \]

(e) Now, if $V_{icm} = 0.1V$, 
- $V_{s1} = V_{s2} = V_c - V_{GS} = 0.1V - 0.7V$
  \[ = -0.6V \]

Since $I$ is a constant current source, $I_{D1}$ and $I_{D2}$ remain at 0.1mA.

This means that $V_{D1}$ and $V_{D2}$ are still 0V.

(d) $V_{ICM} = -0.1V$
- $V_{S1} = V_{S2} = V_G - V_{GS} = 0.1V - 0.7$
  \[ = -0.8V \]

Still, $I_{D1} = I_{D2} = 0.1mA$
- $V_{D1} = V_{D2} = 0V$

(e) $V_{CM}(\text{max}) = V_{DD} - I_D R_D - V_{ov} + V_{GS}$
  \[ = 1 - (0.1mA)(10K) - 0.2 + 0.7 = +0.5V \]

(f) $V_{S}(\text{min}) = - V_{SS} + V_{CS}(\text{min})$
  \[ = -1 + 0.2 = -0.8V \]
- $V_{CM}(\text{min}) = V_S(\text{min}) + V_{GS} = -0.8V + 0.7V$
  \[ = -0.1V \]
2. For the differential amplifier of Problem 1, let $V_{G2} = 0$ and $V_{G2} = V_{id}$. Find the value of $V_{id}$ that corresponds to each of the following situations:

(a) $i_{D1} = i_{D2} = 0.1 mA$; (b) $i_{D1} = 0.15 mA$ and $i_{D2} = 0.05 mA$; (c) $i_{D1} = 0.2 mA$ and $i_{D2} = 0$ (Q2 just cuts off); (d) $i_{D1} = 0.05 mA$ and $i_{D2} = 0.15 mA$; (e) $i_{D1} = 0$ (Q1 just cuts off) and $i_{D2} = 0.2 mA$. For each case, find $v_{id}$, $v_{D1}$, $v_{D2}$, and $(v_{D2} - v_{D1})$.

\[
V_{OV} = \frac{2I_D}{\sqrt{\kappa_p (W/L) \cdot 0.4 mA / V^2}} = 0.2 V
\]

(a) $V_{GS} = V_{OV} + V_t = 0.2 V + 0.5 V = 0.7 V$
$V_s = V_G - V_{GS} = 0 - 0.7 V = -0.7 V$
$V_{D1} = V_{D2} = V_{DD} - i_{D1}R_D = 1.0 V - 0.1 mA$
$(10 k\Omega) = 0 V$
$V_{D2} - V_{D1} = 0 V$

(b) For $i_{D1} = 0.15 mA$, $i_{D2} = 0.05 mA,$

\[
i_{D1} = \frac{l}{2} + \frac{l}{V_{OV}} \cdot \frac{v_{id}}{2} \rightarrow v_{id} = \left(\frac{2i_{D1}}{l} - 1\right) \cdot V_{OV}
\]

\[
v_{id} = \left[\frac{2(0.15 mA)}{0.2 mA} - 1\right] (0.2 V) = 0.1 V
\]

$V_{GS1} = \frac{2(0.15 mA)}{\sqrt{0.4 mA / V^2}} + 0.5 V = 0.745 V$

$V_s = V_G - V_{GS1} = 0.1 V - 0.745 V = -0.645 V$

$V_{D1} = V_{DD} - i_{D1}R_D = 1.0 V - 0.15 mA (10 k\Omega)$
$= -0.5 V$

$V_{D2} = 1.0 V - (0.05 mA) (10 k\Omega) = + 0.5 V$

$V_{D2} - V_{D1} = 1.0 V$

(c) $i_{D1} = 0.2 mA$, $i_{D2} = 0$:

\[
V_{G1} = v_{id} = \sqrt{2} \cdot V_{OV} = 1.414 (0.2 V) = 0.283 V
\]

$V_{GS} = \frac{2(0.2 mA)}{\sqrt{0.4 mA / V^2}} + 0.5 V = 0.783 V$

$V_s = V_G - V_{GS} = 0.283 V - 0.783 V = -0.5 V$

$V_{D1} = 1.0 V - (0.2 mA) (10 k\Omega) = -1.0 V$

$V_{D2} = + 1.0 V$

$V_{D2} - V_{D1} = 2.0 V$

(d) $i_{D1} = 0.05 mA$

\[
i_{D2} = 0.05 mA \quad \text{opposite case of (b)}
\]

For example,

\[
v_{id} = \left[\frac{2(0.05 mA)}{0.2 mA} - 1\right] (0.2 V) = -0.1 V
\]

$V_{GS} = \frac{2(0.05 mA)}{\sqrt{0.4 mA / V^2}} + 0.5 V = 0.641 V$

$V_s = V_G - V_{GS} = -0.1 V - 0.641 V = -0.741 V$

$V_{D1} = 1.0 V - (0.05 mA) (10 k\Omega) = + 0.5 V$

$V_{D2} = 1.0 V - (0.05 mA) (10 k\Omega) = -0.5 V$

$V_{D2} - V_{D1} = -1.0 V$

(e) $i_{D1} = 0$ mA, $i_{D2} = 0.2$ mA is the opposite of (c):

\[
v_{id} = -\sqrt{2} (V_{OV}) = -\sqrt{2} (0.2 V) = -0.283 V
\]

For $i_{D2} = 0.2$ mA, $V_{GS2} = 0.783 V$, so that

$V_s = -0.783 V$

$V_{D1} = 1.0 V$

$V_{D2} = -1.0 V \rightarrow V_{D2} - V_{D1} = -2 V$

The results are summarized in the following table:

<table>
<thead>
<tr>
<th>Case</th>
<th>$V_{id}$ (V)</th>
<th>$i_{D1}$ (mA)</th>
<th>$i_{D2}$ (mA)</th>
<th>$V_s$ (V)</th>
<th>$V_{D1}$ (V)</th>
<th>$V_{D2}$ (V)</th>
<th>$V_{D2} - V_{D1}$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
<td>-0.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(b)</td>
<td>0.1</td>
<td>0.15</td>
<td>0.05</td>
<td>-</td>
<td>-0.645</td>
<td>-0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>(c)</td>
<td>0.283</td>
<td>0.2</td>
<td>0</td>
<td>0.5</td>
<td>-0.783</td>
<td>-1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>(d)</td>
<td>-0.1</td>
<td>0.05</td>
<td>0.15</td>
<td>-</td>
<td>0.5</td>
<td>-0.5</td>
<td>-1.0</td>
</tr>
<tr>
<td>(e)</td>
<td>-0.283</td>
<td>0</td>
<td>0.2</td>
<td>-</td>
<td>1.0</td>
<td>-1.0</td>
<td>-2.0</td>
</tr>
</tbody>
</table>

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3. Consider the differential amplifier specified in Problem 1 with G2 grounded and \( V_{G1} = V_{id} \). Let \( V_{id} \) be adjusted to the value that causes \( i_{D1} = 0.11 \text{mA} \) and \( i_{D2} = 0.09 \text{mA} \). Find the corresponding values of \( V_{GS2} \), \( V_S \), \( V_{GS1} \), and hence \( V_{id} \). What is the difference output voltage \((V_{D2} - V_{D1})\)? What is the voltage gain \((V_{D2} - V_{D1})/V_{id}\)? What value of \( V_{id} \) results in \( i_{D1} = 0.09 \text{mA} \) and \( i_{D2} = 0.11 \text{mA} \)?

\[
V_{G1} = V_{id} i_{D1} = 0.11 \text{ mA} \\
V_{G2} = 0 \quad i_{D2} = 0.09 \text{ mA} \\
I_D = \frac{1}{2} \frac{k'}{n} \frac{W}{L} (V_{GS} - V) ^2
\]

For \( Q_1 \):

\[
0.11 \text{ m} = \frac{1}{2} 5 m (V_{GS1} - 0.5)^2 \\
\rightarrow V_{GS1} = 0.71 \text{ V}
\]

For \( Q_2 \):

\[
0.09 \text{ m} = \frac{1}{2} 5 m (V_{GS2} - 0.5)^2 \\
\rightarrow V_{GS2} = 0.69 \text{ V} \\
V_S = -V_{GS2} = -0.69 \text{ V} \\
v_{id} = V_S + V_{GS1} = -0.69 + 0.71 = 0.02 \text{ V}
\]

\[
V_{D2} - V_{D1} = 10 \text{ k} \Omega \ (i_{D1} - i_{D2}) \\
= 10 \text{ k} \Omega \ (0.11 - 0.09) \text{ m} \\
= 0.2 \text{ V}
\]

thus

\[
\frac{V_{D2} - V_{D1}}{V_{id}} = \frac{0.2}{0.02} = 10
\]

when \( i_{D1} = 0.09 \text{ mA} \) and \( i_{D2} = 0.11 \text{ mA} \)

is the reverse condition from the case we just studied, thus \( v_{id} = -0.02 \text{ V} \)
4. Design the circuit shown below to obtain a dc voltage of +2V at each of the drains of Q1 and Q2 when \( V_{G1}=V_{G2}=0 \). Operate all transistors at \( V_{OV}=0.2 \) V and assume that for the process technology in which the circuit is fabricated, \( V_{th}=0.5 \) V and \( k_n'=250 \mu A/V^2 \). Neglect channel-length modulation. Determine the values of \( R \), \( R_D \), and the W/L ratios of Q1, Q2, Q3, and Q4. What is the input common-mode range for your design?

\[
\begin{align*}
V_{GS} &= V_{in} + V_{OV} = 0.5 \ V + 0.2 \ V = 0.7 \ V \\
V_{D4} &= V_{G4} = -V_{SS} + V_{GS} = -1.2 \ V + 0.7 \ V \\
&= -0.5 \ V \\
R &= \frac{V_{DD} - V_{D4}}{0.1 \ mA} = \frac{1.2 \ V - (-0.5 \ V)}{0.1 \ mA} = 17 \ k\Omega \\
R_D &= \frac{V_{DD} - V_{D4}}{0.4 \ mA / 2} = \frac{1.2 \ V - 0.2 \ V}{0.2 \ mA} = 5 \ k\Omega \\
\left( \frac{W}{L} \right)_1 &= \left( \frac{W}{L} \right)_2 = 0.4 \ mA [k_n' \ V^2] \\
&= 0.2 \ mA [(0.25 \ mA / V^2)(0.2 V)^2]^{-1} = 20 \\
\left( \frac{W}{L} \right)_3 &= 0.4 \ mA [0.01 \ mA]^{-1} = 10 \\
\left( \frac{W}{L} \right)_4 &= 0.1 \ mA [0.01 \ mA]^{-1} = 10 \\
V_{Cw(mn)} &= V_{in} + V_{D4} - (I/2)R_D \\
&= 0.5 \ V + 1.2 \ V - (0.1 \ mA)(5 \ k\Omega) = 0.7 \ V \\
V_{Cw(min)} &= -V_{SS} + V_{or3} + V_{in} + V_{OV} \\
&= -1.2 \ V + 0.2 \ V + 0.5 \ V + 0.2 \ V = -0.3 \ V
\end{align*}
\]

5. Design a MOS differential amplifier to operate from \( \pm 1 \) V power supplies and dissipate no more than 2mW in the equilibrium state. The differential voltage gain \( A_d \) is to be 5 V/V and the output common-mode dc voltage is to be 0.5V. (Note: This is the dc voltage at the drains). Assume \( k_n'=400 \mu A/V^2 \) and neglect the Early effect. Specify the required values of \( I \), \( R_D \), and W/L.

\[
\begin{align*}
+1 \ V \ & \text{supplies not more than } 2 \text{mW } A_d = 5 \text{V/V} \\
V_0 &= 0.5 \ V \ K_n' = \mu_n C_{ox} = 0.4 \ mA/V^2 \\
L &= \frac{2 \ mA}{1 \ V - (-1 \ V)} = 1 \ mA \\
R_D &= \frac{1 \ V - 0.5 \ V - 0.5 \ V}{1/2 I} = 1 \ k\Omega \\
g_n &= \frac{A_d}{R_D} = \frac{5 \ V/V}{1 \ k\Omega} = 5 \ mA/V \\
V_{OV} &= \frac{I}{g_n} = \frac{1 \ mA}{5 \ mA/V} = 0.2 \ V \\
W \ = \ L &= 2(I/2) / (k_n' V_{OV}^2) \\
&= 1 \ mA / (0.4 \ mA/V^2 - (0.2 \ V)^2) = 62.5 \\
\text{Because we picked } L=1 \ mA \text{ THIS IS THE SOLUTION WITH THE HIGHEST ALLOWABLE POWER. THIS SOLUTION WILL ALSO THEREFORE HAVE THE WIDEST RANGE OF DIFFERENTIAL MODE OPERATION. AN INFINITE NUMBER OF OTHER SOLUTIONS EXIST.}
\end{align*}
\]
6. An NMOS differential amplifier is operated at a bias current $I$ of 0.4mA and has a W/L ratio of 32, $k_n' = \mu_n C_{ox} = 200\mu A/V^2$, $V_A = 10V$, and $R_D = 5k\Omega$. Find $V_{OV} = (V_{GS} - V_t)$, $g_m$, $r_o$, and $A_d$.

$$I = 0.4\ mA \quad \text{W/L} = 32 \quad k_n' = \mu_n C_{ox}$$

$$V_A = 10\ V \quad R_D = 5\ k\Omega$$

$$V_{OV} = \sqrt{\frac{I}{k_n' \frac{W}{L}}} = \sqrt{0.4 \cdot (0.2 \cdot 32)} = 0.25\ V$$

$$g_m = \frac{I}{V_{OV}} = \frac{0.4\ mA}{0.25\ V} = 1.6\ mA/V$$

$$r_o = \frac{V_A}{I_D} = \frac{10\ V}{0.2\ mA} = 50\ k\Omega$$

$$A_d = g_m \left( R_D \parallel r_o \right) = 1.6 \ (5 \parallel 50) = 7.3\ V/V$$

7. An active-loaded NMOS differential amplifier operates with a bias current $I$ of 100µA. The NMOS transistors are operated at $V_{OV} = 0.2V$ and the PMOS dives at $|V_{OV}| = 0.3V$. The Early voltages are 20V for the NMOS and 12V for the PMOS transistors. Find $G_m$, $R_o$ (output R), and $A_d$. For what value of load resistance is the gain reduced by a factor of 2?

$$I_{D1} = I_{D2} = I_{N3} = I_{D4} = I = \frac{100\ \mu A}{2} = 50\ \mu A$$

$$g_m = \frac{I_D}{V_{OV}} = \frac{50\ \mu A}{0.2\ V} = 0.5\ mA/V$$

$$r_o = \frac{V_{A}}{I_D} = \frac{20}{0.05\ mA} = 400\ k\Omega$$

$$r_o = \frac{V_{AD}}{I_D} = \frac{12\ V}{0.05\ mA} = 240\ k\Omega$$

$$R_o = r_o2 \parallel r_o4 = 400\ k\parallel 240\ k = 150\ k\Omega$$

$$A_d = G_m R_o = (0.5\ mA/V) \left( 150 \ k\Omega \right) = 75 \ V/V$$

Gain will be reduced by a factor of 2 if $R_L = R_o = 150\ k\Omega$