Problem 1 (20 pts)

You need to transmit data over an optical link of 100 km with fiber loss of 0.2 dB/km. The link has five splices with 0.05 dB loss per splice and two connectors with 0.2 dB per connector. The receiver sensitivity is 20 $\mu$W. Express the minimum transmitter power in both mW and dBm.

You can express the loss budget of the system (in dB) using

\[ P_{tr, dBm} - P_{rec, dBm} = -\gamma L + L_{fixed}, \]

where $L_{fixed}$ represents the fixed losses in the system (i.e. losses that do not depend upon fiber length). Remember that $\gamma = -0.2 \text{ dB/km} < 0$ for loss.

From the problem statement, the fixed losses add up to

\[ L_{fixed} = 2 \times L_{connector} + 5 \times L_{splice} = 2 \times 0.2 \text{ dB} + 5 \times 0.05 \text{ dB} = 0.65 \text{ dB}, \]

and the fiber loss

\[ -\gamma L = 0.2 \text{ dB/km} \times 100 \text{ km} = 20 \text{ dB}. \]

Now, we need to express the receiver sensitivity in dBm, which we do by

\[ P_{rec, dBm} = 10 \log \left( \frac{20 \ \mu W}{1 \text{ mW}} \right) = -17.0 \text{ dBm}. \]

Solving for the transmitter power, we get

\[ P_{tr, dBm} = P_{rec, dBm} - \gamma L + L_{fixed} = -17.0 \text{ dBm} + 20 \text{ dB} + 0.65 \text{ dB} = 3.66 \text{ dBm}. \]

Expressed in mW, $P_{tr} = 2.32 \text{ mW}$. 
Problem 2 (40 pts)

Consider an optical fiber of 50 μm diameter, core index \( n_1 = 1.5 \), and cladding index \( n_2 = 1.49 \) for operation at \( \lambda = 1.31 \) μm.

a) What is the numerical aperture (NA) of this fiber?

The numerical aperture is defined as

\[
NA \equiv \sin \theta = \sqrt{n_1^2 - n_2^2}.
\]

Plugging in the numbers, we get

\[
NA = \sqrt{1.5^2 - 1.49^2} = 0.173.
\]

b) How many modes does this fiber support?

We first have to calculate the V number for the fiber

\[
V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} N.A.
\]

Note that \( 2a \) is the diameter. Plugging in the numbers, we get

\[
V = \frac{\pi \times 50 \, \mu m}{1.31 \, \mu m} \times 0.173 = 20.7.
\]

Since \( V > 10 \), we can use \( V^2/2 \) for the number of modes, which comes out to be 215.

c) What would be the pulse spread due to modal dispersion over a distance of 10 km?

Pulse spread due to modal dispersion is described by

\[
\Delta \tau_{\text{modal}} = \frac{n_1 L}{c} \Delta,
\]

where

\[
\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.5 - 1.49}{1.5} = 6.67 \times 10^{-3}.
\]

Now, plugging in the rest of the numbers,

\[
\Delta \tau_{\text{modal}} = \frac{1.5 \times 10 \, \text{km}}{3 \times 10^5 \, \text{km/sec}} \times 6.67 \times 10^{-3} = 333 \, \text{ns}.
\]

d) What would the maximum fiber diameter need to be for the fiber to operate with a single mode?

For a single-mode fiber, we must have the condition that \( V \leq 2.405 \). Solving for \( 2a \),

\[
V = \frac{2\pi a}{\lambda} NA = 2.405 \Rightarrow 2a = \frac{2.405 \lambda}{\pi NA} = \frac{2.405 \times 1.31}{\pi \times 0.173} \, \mu m = 5.80 \, \mu m.
\]
**Problem 3** (40 pts)

For an optical communications system, the transmitter and receiver operate at 2.5 Gb/sec NRZ ($B_{NRZ}$) at a central wavelength of 1550 nm, using a laser with a spectral linewidth of $\Delta \lambda = 0.05$ nm. The fiber has a dispersion parameter of $M = -20$ ps/nm-km.

a) Calculate the pulse spread per unit distance (ps/km).

The pulse spread due to chromatic dispersion is written

$$\Delta \tau_{\text{chrom}} = -M \times L \times \Delta \lambda \Rightarrow \Delta \left( \frac{\tau}{L} \right)_{\text{chrom}} = -M \times \Delta \lambda = 20 \text{ ps/nm} \cdot \text{km} \times 0.05 \text{ nm} = 1 \text{ ps/km}.$$

b) What is the maximum length of fiber that allows the stated system bit rate?

*We have to go back to the expression for dispersion-limited bit-rate*

$$B_{NRZ} = 2 \times f_{3dB,\text{elec}} = \frac{0.7}{\Delta \tau} = \frac{0.7}{\Delta (\tau/L)_{\text{chrom}} \times L}.$$

Now, we solve for the unknown $L$,

$$L = \frac{0.7}{\Delta (\tau/L)_{\text{chrom}} \times B_{NRZ}} = \frac{0.7}{1 \text{ ps/km} \times 2.5 \text{ Gb/sec}} = 280 \text{ km}.$$

c) If the system were to operate at 10 Gb/sec NRZ, then what would be the required optical bandwidth $f_{3dB,\text{optical}}$? Hint: you don’t need fiber length or $\Delta \lambda$ for this calculation.

*We have to go back to the relationships among the different bandwidths. We know that*

$$f_{3dB,\text{electrical}} = \frac{\sqrt{2}}{2} \times f_{3dB,\text{optical}}$$

*and that*

$$B_{NRZ} = 2 \times f_{3dB,\text{electrical}} = \sqrt{2} f_{3dB,\text{optical}},$$

*so that*

$$f_{3dB,\text{optical}} = \frac{B_{NRZ}}{\sqrt{2}} = 7.07 \text{ GHz}.$$

d) Given the optical bandwidth of part c), then what would be the optical frequency spread in terms of $\Delta \lambda$? What would be the maximum fiber length?

*The 3 dB optical bandwidth is just $\Delta \nu$, and we can use the relationship*

$$\frac{\Delta \nu}{\nu} = \frac{\Delta \lambda}{\lambda}$$

*to solve for $\Delta \lambda$. Doing so, we get*

$$\Delta \lambda = \frac{\Delta \nu}{\nu} \times \lambda = \frac{7.07 \text{ GHz}}{193.5 \text{ THz}} \times 1550 \text{ nm} = 0.057 \text{ nm}.$$
We have to calculate the new wavelength spread, which is done by

\[ \Delta \lambda^2 = \Delta \lambda^2_{\text{source}} + \Delta \lambda^2_{\text{modulation}} = (0.05 \text{ nm})^2 + (0.057 \text{ nm})^2 \Rightarrow \Delta \lambda = 0.076 \text{ nm}. \]

Using this new wavelength spread, we can recalculate the dispersion limited distance

\[
L = \frac{0.7}{\Delta (\tau/L)_{\text{chrom}} \times B_{\text{NRZ}}} = \frac{0.7}{20 \text{ ps/nm} \cdot \text{km} \times 0.076 \text{ nm} \times 10 \text{ Gb/sec}} = 46 \text{ km}.
\]