4.5. Receiver sensitivity

The bit-error rate, or BER, is the probability of incorrect identification of an incoming bit by the data recovery unit of the receiver. For example, a BER of $2 \times 10^{-6}$ corresponds to one incorrect bit in 500,000. In optical communications, we want $\text{BER} \leq 10^{-9}$, with is the raw BER, or the BER in the absence of error detection/correction coding. The receiver sensitivity is the average power $P_{\text{rec}}$ required by the receiver to achieve $\text{BER} = 10^{-9}$.

The figure shows the signal received by the data recovery unit. The signal fluctuates due to noise, and the job of the decision unit is to sample the signal at the decision instant $t_D$, which is in the center of the bit slot as determined by the clock recovery circuit. A threshold $I_D$ is determined for which sampled values $I > I_D$ corresponds to a “1” bit and values $I < I_D$ correspond to a “0” bit. An error occurs if these recovered bits do not match the corresponding transmitted bits.

4.5.1. Bit-error rate

The error probability is defined

$$\text{BER} = p(1)p(0|1) + p(0)p(1|0)$$

$$= \frac{1}{2} \left[ P(0|1) + P(1|0) \right],$$

where $p(0) = p(1) = 1/2$ over a long time average. Here, $p(0)$ is the probability of “0” being transmitted, $p(1)$ is the probability of “1” being transmitted, $p(0|1)$ is the probability of deciding “0” when “1” was intended, and $p(1|0)$ is the probability of deciding “1” when “0” was intended. Each sampled value of $I$ has Gaussian PDF with variance

$$\sigma^2 = \sigma_s^2 + \sigma_T^2,$$

but the variance differs for “1” and “0” bits as the signal levels $I_1$ and $I_0$ are different. We denote these variances by $\sigma_1^2$ about “1” and $\sigma_0^2$ about the “0” bit.
The conditional probabilities are evaluated by

\[
P(0|1) = \frac{1}{\sigma_1 \sqrt{2\pi}} \int_{-\infty}^{I_D} e^{-(I-I_1)^2/2\sigma_1^2} dI
\]

\[
= \frac{1}{2} \text{erfc}\left(\frac{I_1 - I_D}{\sigma_1 \sqrt{2}}\right)
\]

\[
P(1|0) = \frac{1}{\sigma_0 \sqrt{2\pi}} \int_{I_D}^{\infty} e^{-(I-I_0)^2/2\sigma_0^2} dI
\]

\[
= \frac{1}{2} \text{erfc}\left(\frac{I_D - I_0}{\sigma_0 \sqrt{2}}\right)
\]

The complementary error function is defined

\[
\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-y^2} dy.
\]

Now, the bit-error rate can be written

\[
\text{BER} = \frac{1}{4} \left[ \text{erfc}\left(\frac{I_1 - I_D}{\sigma_1 \sqrt{2}}\right) + \text{erfc}\left(\frac{I_D - I_0}{\sigma_0 \sqrt{2}}\right) \right]
\]

Now the question arises as to the optimal value of the decision threshold \(I_D\). The optimal value occurs when

\[
\frac{I_1 - I_D}{\sigma_1} = \frac{I_D - I_0}{\sigma_0} \equiv Q
\]

or

\[
I_D = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}.
\]

Note that when thermal noise dominates, \(\sigma_0 = \sigma_1\), and \(I_D = (I_1 + I_0)/2\). Shot noise depends on the current levels and is larger for 1 bits. Using the optimal \(I_0\),

\[
\text{BER} = \frac{1}{2} \text{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx e^{-Q^2/2Q\sqrt{2\pi}}
\]

where the approximation is valid when \(Q > 3\), and

\[
Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}.
\]

The \(\text{BER} \approx 10^{-9}\) when \(Q=6\), and \(< 10^{-12}\) for \(Q > 7\).
4.5.2. Minimum received power

If we assume that $P_0 = 0$ for the received “0” bits, then $I_0 = 0$. For the “1” bits,

$$I_1 = MRP_1 = 2MR \overline{P}_{\text{rec}},$$

where $\overline{P}_{\text{rec}} = (P_1 + P_0)/2 = P_1/2$ is the average received power. Remember that $M = 1$ for a pin detector. The RMS noise currents are

$$\sigma_1 = \sqrt{\sigma_s^2 + \sigma_T^2} \quad \text{and} \quad \sigma_0 = \sigma_T,$$

where the shot and thermal noise contributions are given by

$$\sigma_s^2 = 2qM^2F_AR(2\overline{P}_{\text{rec}})\Delta f$$

$$\sigma_T^2 = \left(\frac{4k_BT}{R_L}\right)F_N\Delta f.$$

Now, $Q$ can be evaluated

$$Q = \frac{I_1}{\sigma_1 + \sigma_0} = \frac{2MR \overline{P}_{\text{rec}}}{\sqrt{\sigma_s^2 + \sigma_T^2 + \sigma_T^2}}.$$

Note that $\overline{P}_{\text{rec}}$ shows up in $\sigma_s^2$ as well.

The receive sensitivity is simply the average received power needed to obtain a given BER, where the BER is directly related to $Q$. Therefore, solving for $\overline{P}_{\text{rec}},$

$$\overline{P}_{\text{rec}} = \frac{Q}{R} \left(qFA\Delta f + \frac{\sigma_T}{M}\right).$$

If we assume for the moment that $\sigma_T$ dominates, then

$$Q = \frac{MR \overline{P}_{\text{rec}}}{\sigma_T} \Rightarrow \overline{P}_{\text{rec}} = \frac{\sigma_T Q}{MR},$$

which shows the APD advantage in this limit. Since $\sigma_T \propto \Delta f$, and $\Delta f \sim B/2$, then the receiver sensitivity increases (which is bad) as $\sqrt{B}$. For PIN detectors, $\overline{P}_{\text{rec}} \sim -30$ dBm.

If we assume that shot noise, or $\sigma_s$, dominates, then

$$Q = \frac{2MR \overline{P}_{\text{rec}}}{\sigma_s} = \frac{2R \overline{P}_{\text{rec}}}{\sqrt{2qFA R(2\overline{P}_{\text{rec}})\Delta f}}$$

$$= \frac{\sqrt{RP_{\text{rec}}}}{qFA\Delta f},$$

and the receiver sensitivity

$$\overline{P}_{\text{rec}} = \frac{qFA\Delta f Q^2}{R}.$$