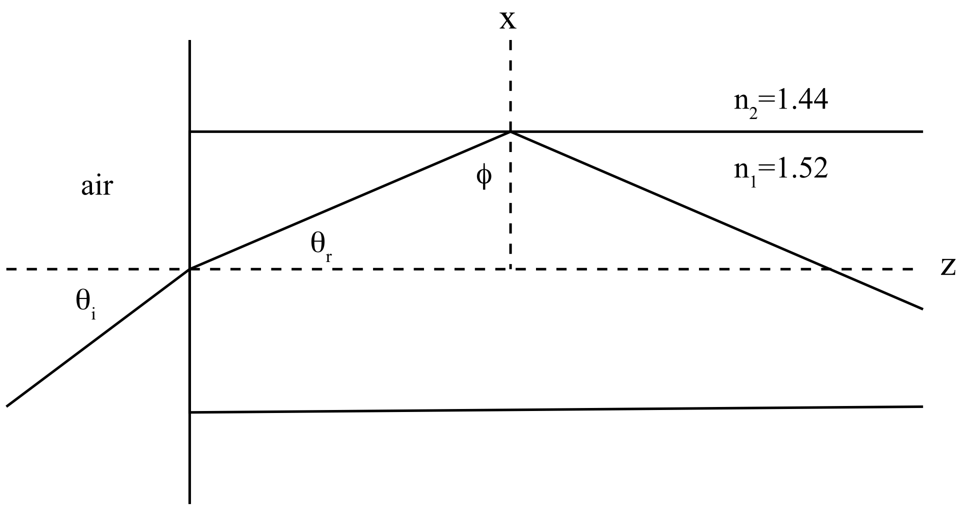
ECE 6440 - Photonic Microsystems
Midterm Exam I Spring 2011

Problem 1 (25 points)

Consider the following slab dielectric interface:



a) What is the minimum angle ϕ for total internal reflection?

The critical angle is defined by

$$n_1 \sin \phi_{crit} = n_2 \sin(\pi/4)$$
$$\rightarrow \phi_{crit} = \arcsin\left(\frac{n_2}{n_1}\right) = \arcsin\left(\frac{1.44}{1.52}\right) = 71.3^o$$

b) What is the maximum angle θ_i for total internal reflection? What is the numerical aperture?

The internal angle θ_r is related to the critical angle by

$$\theta_r = \frac{\pi}{4} - \phi_{crit},$$

and the external and internal coupling angles are related by Snell's Law

$$\sin \theta_i = n_1 \sin \theta_r = n_1 \sin\left(\frac{\pi}{4} - \phi_{crit}\right) = n_1 \cos \phi_{crit}.$$

Using the fact that $\sin \phi_{crit} = n_2/n_1$, this can be rewritten

$$\sin \theta_i = n_1 \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = \sqrt{n_1^2 - n_2^2}.$$

Plugging in the numbers

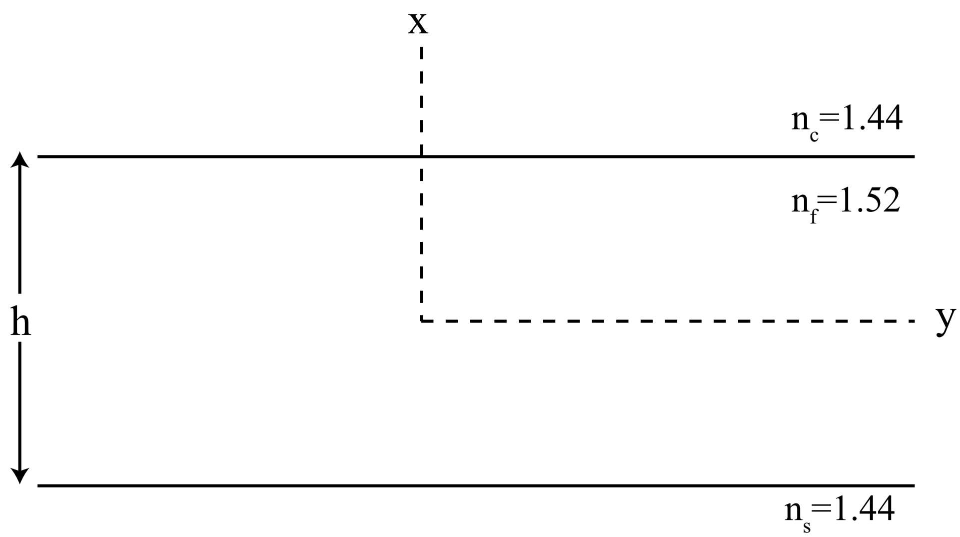
$$\sin \theta_i = NA = \sqrt{1.52^2 - 1.44^2} = 0.487$$

and

$$\theta_i = 29.1^o.$$

Problem 2 (25 points)

Consider the following symmetric slab dielectric waveguide:



a) If the waveguide thickness is $h = 0.7\ \mu$m, then what is the minimum value of λ_f for which the waveguide is single mode?

TM modes always cut off before TE modes. The maximum value of V for which the waveguide can be single mode is $V \sim 3.1$. The V number is defined as

$$V = \frac{2\pi h}{\lambda_f}\sqrt{n_f^2 - n_s^2} = \frac{2\pi \times 0.7\ \mu\text{m}}{\lambda_f}\sqrt{1.52^2 - 1.44^2} = \frac{2.14}{\lambda_f}\ \mu\text{m}.$$

In order for $V < 3.1$, the minimum value of λ_f is $0.690\ \mu$m.

b) At the above wavelength and with $h = 5\ \mu$m, how many modes does the waveguide support (both TE and TM)?

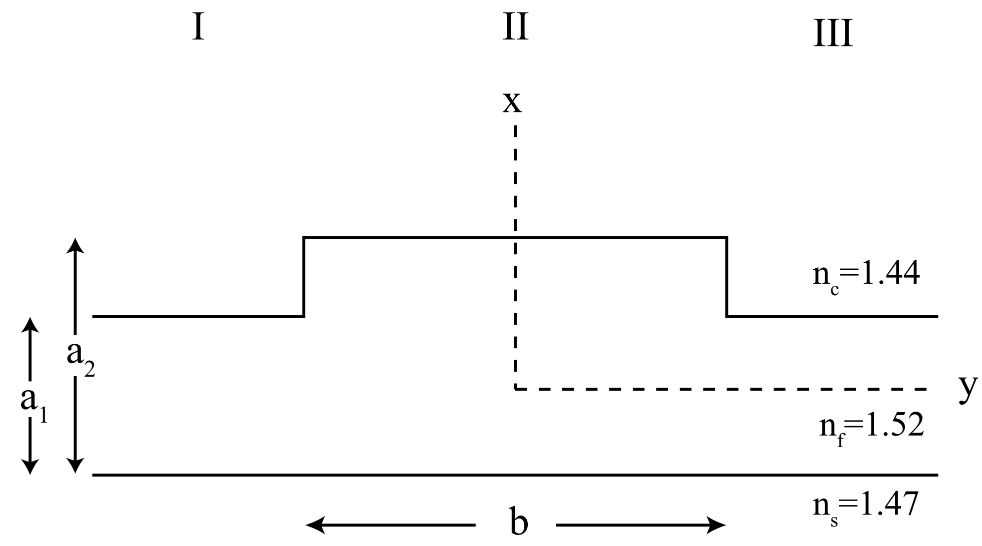
Again, we go back to the V number.

$$V = \frac{2\pi h}{\lambda_f}\sqrt{n_f^2 - n_s^2} = 22.1.$$

From the graph, $V = 22.1$ gives 8 modes. Assuming that the graph represents both TE and TM modes, then the total number is 16.

Problem 3 (50 points)

Consider the following ridge dielectric waveguide:



The free-space wavelength is $\lambda_f = 1.0$ μm, $a_1 = 0.6$ μm, $a_2 = 1$ μm, and $b = 4$ μm. The electric field points along the y-direction.

a) Solve for the effective indices in regions I, II, and III.

We start by calculating the V number in each region (I and III being the same).

$$V_I = \frac{2\pi \times 0.6\ \mu\text{m}}{1.0\ \mu\text{m}}\sqrt{1.52^2 - 1.47^2} = 1.46$$

$$V_{II} = \frac{2\pi \times 1.0\ \mu\text{m}}{1.0\ \mu\text{m}}\sqrt{1.52^2 - 1.47^2} = 2.43$$

Since all three regions are also asymmetric, we also have to calculate a (for TE slab modes):

$$a = \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2} = \frac{1.47^2 - 1.44^2}{1.52^2 - 1.47^2} = 0.58.$$

Looking at the graph, we can get the values of b for each region: $b_I = 0.2223, b_{II} = 0.4877$. Knowing that

$$b = \frac{n_{eff}^2 - n_s^2}{n_f^2 - n_s^2},$$

we can get n_{eff} for each region: $n_{eff,I} = 1.48126$, and $n_{eff,II} = 1.49459$.

Note that each of the three regions is single mode.

b) How many modes does this waveguide support? What are the effective indices?

We now calculate the V number for the new effective waveguide,

$$V = \frac{2\pi \times 4\ \mu\text{m}}{1.0\ \mu\text{m}}\sqrt{n_{eff,II}^2 - n_{eff,I}^2} = 5.01.$$

Since this is a symmetric waveguide (even for TM), $a = 0$.

From the normalized eignevalue plot (assuming that it is correct for TM slab modes), we can see that $V = 5.0$ gives us two modes ($b = 0.803, 0.278$). Their effective indices are 1.49197 and 1.48498. Calculating correctly for the TM mode, we get indices 1.49195 and 1.48494.

c) What, if anything, changes if $\lambda_f = 1.55$ μm?

We get n_{eff} for each region: $n_{eff,I} = 1.47263$, and $n_{eff,II} = 1.48306$; these indices decrease.

The rib now has $V = 2.85$, which is single mode with $n_{eff} = 1.47894$.

The rib will support a single mode, and the mode size should be larger than the fundamental mode of the waveguide at the shorter wavelength.