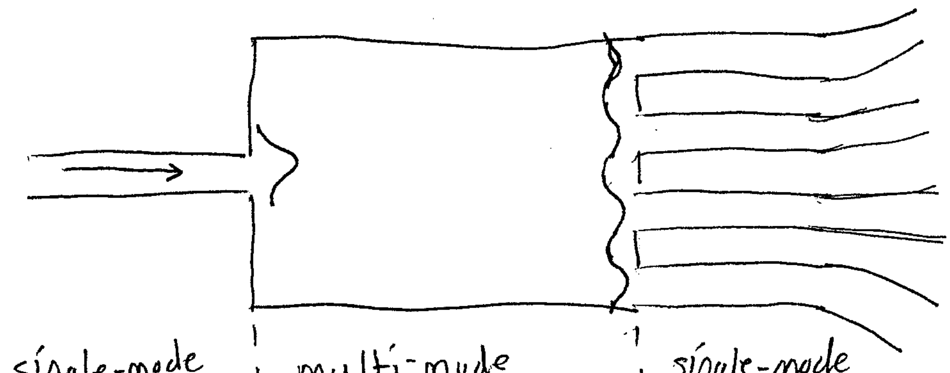
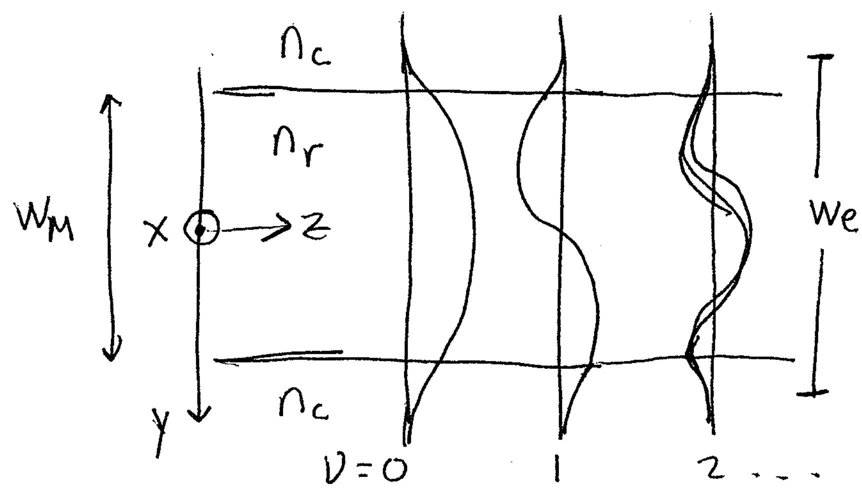
~~[illegible]~~

Multi-mode interference couplers



multi-mode region



m modes $\nu = 0, 1, 2, \ldots (m-1)$
lateral wavenumber $k_{y\nu}$ (i.e. κ)
prop. const. β_ν

$$k_{y\nu}^2 + \beta_\nu^2 = \left(\frac{2\pi}{\lambda_0}\right)^2 n_r^2 = k_0^2 n_r^2$$

define an effective width via $k_{y\nu} = \dfrac{(\nu+1)\pi}{W_{e\nu}}$

for high index contrast, $W_e \approx W_M$

more generally,

$$W_{e0} \approx W_M + \left(\frac{\lambda_0}{\pi}\right)\left(\frac{n_c}{n_r}\right)^{2\sigma} \frac{1}{\sqrt{n_r^2 - n_c^2}}$$

assume $W_{e\nu} \sim W_{e0}$ $\quad \sigma = 0$ TE and $\sigma = 1$ TM if we assume $k_{y\nu} \ll k_0 n_r$

$$\beta_\nu = \sqrt{k_0^2 n_r^2 - k_{y\nu}^2} = k_0 n_r \sqrt{1 - \left(\frac{k_{y\nu}}{k_0 n_r}\right)^2}$$

$$\approx k_0 n_r \left[1 - \frac{1}{2}\left(\frac{k_{y\nu}}{k_0 n_r}\right)^2\right]$$

$$= k_0 n_r - \frac{k_{y\nu}^2}{2 k_0 n_r} = k_0 n_r - \frac{(\nu+1)^2 \pi^2}{2 k_0 n_r W_{e\nu}^2}$$

quadratic with mode number

L_π = beat length between two lowest-order modes ($\nu = 0, 1$)

$$= \frac{\pi}{\beta_0 - \beta_1} \approx \frac{4 n_r W_e^2}{3 \lambda_0}$$

$$\Rightarrow \beta_0 - \beta_\nu \approx \frac{\nu(\nu+2)\pi}{3 L_\pi}$$

an input profile $\Psi(y,0)$ can be decomposed into modal field distributions $\Psi_\nu(y)$ in multi-mode region

$$\Psi(y,0) = \sum_{\nu=0}^{m-1} c_\nu \Psi_\nu(y) \qquad \text{ignoring radiation modes}$$

$$\int \Psi(y,0)\, \Psi_{\nu'}^*(y)\, dy = \sum_{\nu=0}^{m-1} c_\nu \underbrace{\int \Psi_\nu(y)\, \Psi_{\nu'}^*(y)\, dy}_{\neq 0 \text{ only for } \nu = \nu'}$$

$$\int \Psi(y,0)\, \Psi_\nu^*(y)\, dy = c_\nu \int \Psi_\nu(y)\, \Psi_\nu^*(y)\, dy$$

$$\Rightarrow c_\nu = \frac{\int \Psi(y,0)\, \Psi_\nu^*(y)\, dy}{\int |\Psi_\nu(y)|^2\, dy} \quad \leftarrow \text{or } \sqrt{\int}\ ?$$

in other words, input will couple into mode spectrum of MM with efficiencies given by c_ν

$$\psi(y,z) = \sum c_\nu \psi_\nu(y)\, e^{j(\omega t - \beta_\nu z)}$$

$$\propto \sum c_\nu \psi_\nu(y)\, e^{j(\beta_0 - \beta_\nu) z}$$

$$= \sum c_\nu \psi_\nu(y)\, e^{j \frac{\nu(\nu+2)\pi}{3L_\pi} z}$$

$\therefore$ output determined by c_ν and phase term

$$\nu(\nu+2) = \begin{cases} \text{even for } \nu \text{ even} \\ \text{odd for } \nu \text{ odd} \end{cases}$$

<u>single "images"</u>

what if $\dfrac{\nu(\nu+2)\pi}{3L_\pi} \cdot z = 2\pi m$?

$$\frac{(\nu+1)(\nu+3)\pi z}{3L_\pi} - \frac{\nu(\nu+2)\pi z}{3L_\pi} = 2\pi m?$$

then $\psi(y,z) = \sum c_\nu \psi_\nu(y) = \psi(y,0)$!

we get exactly the same input back as the output

$$z = 6L_\pi \cdot m / \nu(\nu+2) = \frac{3L_\pi(2m)}{\nu(\nu+2)}$$

ν has to be even

$$\frac{(\nu^2+4\nu+3)\pi z}{3L_\pi} - \frac{(\nu^2+2\nu)\pi z}{3L_\pi} = \frac{(2\nu+3)\pi z}{3L_\pi} = 2\pi m$$

$$z = \frac{6mL_\pi}{2\nu+3}$$

$$z = \frac{3L_\pi(2m)}{\nu(\nu+2)} = \frac{3L_\pi p}{\nu(\nu+2)}$$

let $z = 3pL_\pi$ where $p = 0, 2, 4, \ldots$ (even)

then $\dfrac{\nu(\nu+2)\pi}{3L_\pi} \cdot z = \dfrac{\nu(\nu+2)\pi}{\cancel{3}} \cdot p$ will always $= 2\pi m$ for some m

what if $e^{j\frac{\nu(\nu+2)\pi}{3L_\pi}z} = (-1)^\nu$

then modes alternately add + subtract and we get mirror image

$z = 3(2p+1)L_\pi \qquad (2p+1)\text{ odd}$

$$e^{j\frac{\nu(\nu+2)\pi}{3L_\pi}\cdot 3L_\pi(2p+1)} = e^{j\nu(\nu+2)(2p+1)\pi}$$

ν even, then $e^{j\cdot\text{phase}} = -1$
ν odd, then $e^{j\cdot\text{phase}} = 1$

multiple images

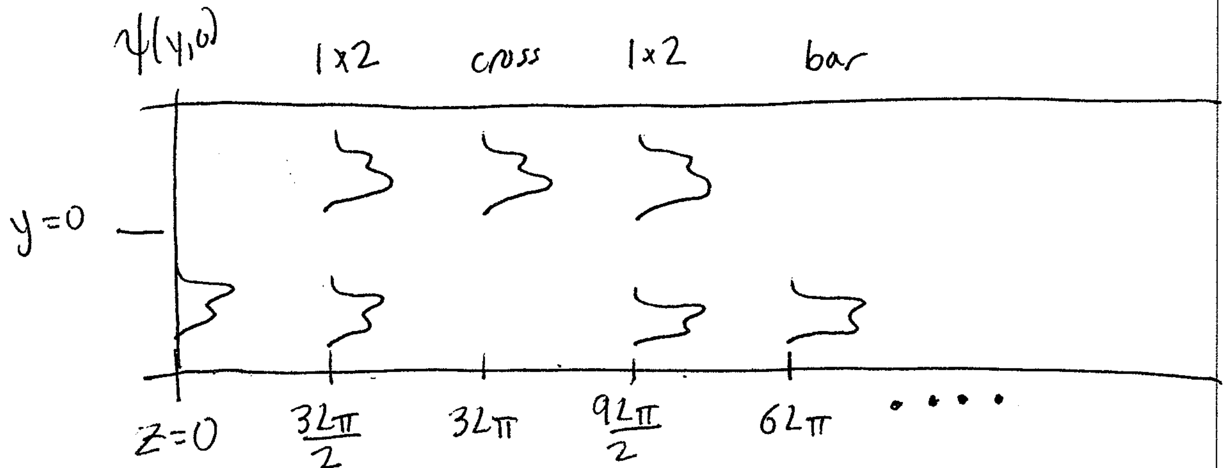
what if $L = \frac{p}{2}(3L_\pi) \qquad p = 1, 3, 5, \ldots$

$$\psi(y,z) = \sum c_\nu \psi_\nu(y) e^{j\nu(\nu+2)p\cdot\pi/2}$$

$$= \sum_{\text{even}} c_\nu \psi_\nu(y) + (-j)^p \sum_{\text{odd}} c_\nu \psi_\nu(y)$$

(-y,0)

$$= \frac{1+(-j)^p}{2}\psi(y,0) + \frac{1-(-j)^p}{2}\psi(-y,0)$$



symmetric excitation (i.e. $c_\nu = 0$ for ν odd)

single images obtained at

$$e^{j\frac{\nu(\nu+2)\pi z}{3L_\pi}} = 1$$

since $\nu(\nu+2)$ guaranteed to be even (and a multiple of 4)

$$z = 3L_\pi(p/4)$$ guarantees phase condition

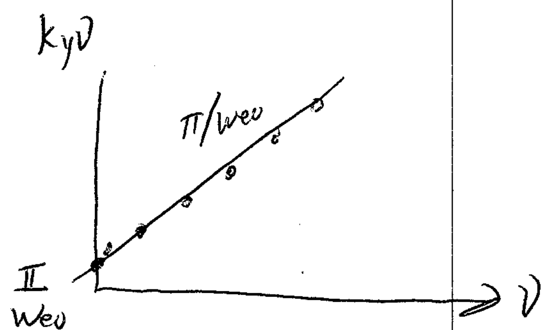
N-fold images obtained at distances

$$z = \frac{p}{N}\left(\frac{3L_\pi}{4}\right)$$

for N-way splitting, need at least $m = N+1$ modes

Homework

1. verify $k_{y\nu} = \frac{(\nu+1)\pi}{\cancel{L_\pi} W_{eo}}$



2. design, sim + opt. 1×2 splitter (MMI)
   insertion loss, split ratio

3. design, sim + opt 2×4 MMI splitter