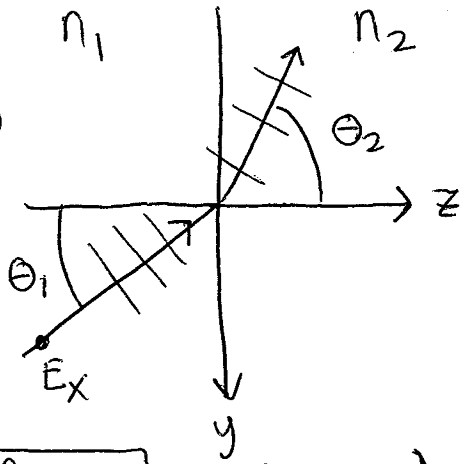
Wave description of TIR

$$E_1 = \hat{x} E_0 e^{-jk_0 n_1 (z\cos\theta_1 - y\sin\theta_1)}$$

$$E_2 = \tau \hat{x} E_0 e^{-jk_0 n_2 (z\cos\theta_2 - y\sin\theta_2)}$$

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

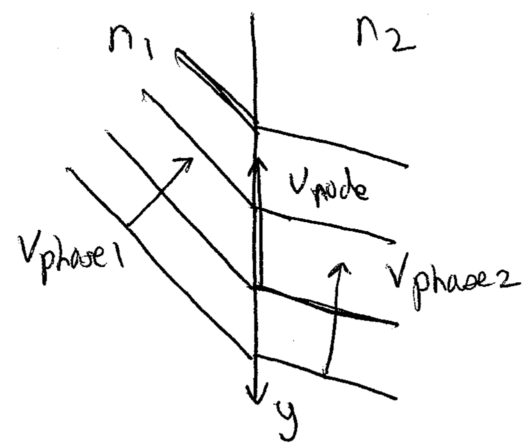
$$\cos\theta_2 = \sqrt{1 - \frac{n_1^2}{n_2^2}\sin^2\theta_1}$$



$$E_2 = \tau \hat{x} E_0 e^{-jk_0 n_2 \left(z\sqrt{1 - \frac{n_1^2}{n_2^2}\sin^2\theta_1} - y\frac{n_1}{n_2}\sin\theta_1 \right)}$$

wave fronts in-phase at interface, i.e. tangential components continuous

($n_2 \sin\theta_2 = n_1 \sin\theta_1$)



phase velocity @ interface

$y\, k_0 n_1 \sin\theta_1 + \omega t = \text{const}$

$$\Rightarrow \frac{dy}{dt} = v_{node} = -\frac{\omega}{k_0 n_1 \sin\theta_1} = \frac{v_{phase1}}{\sin\theta_1} \left(= \frac{v_{phase2}}{\sin\theta_2} \right)$$

$$v_{phase1} = c/n_1$$

at $\theta_1 = \theta_{cr}$, $\cos\theta_2 \to 0$ $\quad \sin\theta_2 \to 1$

$$E_2 = \tau \hat{x}\, e^{jk_0 n_2 y}$$

plane wave travelling // to interface along y-dir

$v_{node} = v_{phase2}$ = min. velocity

for $\theta_1 > \theta_{cr}$ ` $\sqrt{}$ becomes imaginary

$$E_2 = \tau \hat{x} E_0 e^{-k_0 n_2 \sqrt{(n_1/n_2)^2 \sin^2\theta_1 - 1}\, z} e^{jk_0 n_1 \sin\theta_1 y}$$

$$= \tau \hat{x} E_0 e^{-\gamma z} e^{j\beta y}$$

γ = attenuation coefficient (cm^{-1}) $= k_0 n_2 \sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1}$

$\beta = k_0 n_1 \sin\theta_1$ propagation coefficient (rad/m)

E_2 decays exponentially along z and is known as evanescent

E and H are 90° out of phase, reactive power where energy is stored, but no work done

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_2 & 0 & 0 \end{vmatrix} = \hat{y}\frac{\partial E_2}{\partial z} - \hat{z}\frac{\partial E_2}{\partial y}$$

$$= -\gamma \hat{y} E_2 - j\beta \hat{z} E_2$$

$$= -j\omega \vec{B} = -j\omega\mu_0 \vec{H}$$

$$\vec{H} = \underbrace{\frac{\gamma}{j\omega\mu_0} E_2 \hat{y}}_{90^\circ \text{ phase}} + \frac{\beta}{\omega\mu_0} E_2 \hat{z}$$

look @ power $S_z = \frac{1}{2} Re\left[\vec{E} \times \vec{H}^* \cdot \hat{z}\right]$

$$\vec{E} \times \vec{H}^* = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ E_2 & 0 & 0 \\ 0 & -\frac{\gamma}{j\omega\mu_0} E_2^* & \frac{\beta}{\omega\mu_0} E_2^* \end{vmatrix} = \hat{z} \cdot \frac{j\gamma}{\omega\mu_0} |E_2|^2 - \hat{y} \frac{\beta}{\omega\mu_0} |E_2|^2$$

$$S_z = \frac{1}{2} Re\left[\frac{j\gamma}{\omega\mu_0} |E_2|^2\right] = \underline{\underline{0}}$$

Phase shift upon reflection

upon TIR, phase of reflection lags incident
$\Rightarrow$ extra distance light travels into and from low index medium
known as Goos-Hänchen shift

$$\frac{E_r}{E_i} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = |r| e^{j 2\phi}$$

$$= \frac{\alpha - j\beta}{\alpha + j\beta} \qquad \alpha = n_1 \cos\theta_1 \qquad j\beta = n_2 \cos\theta_2$$

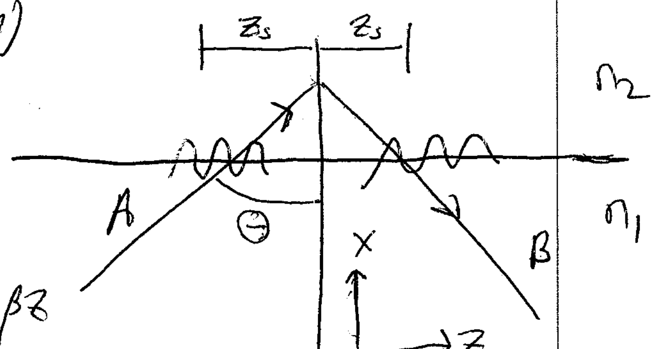
$$2\phi_{TE} = 2\tan^{-1}\left(\frac{-\sqrt{n_1^2 \sin^2\theta_1 - n_2^2}}{n_1 \cos\theta_1}\right) \qquad \theta > \theta_{CR}$$

$$|E_r/E_i| = 1$$

$$2\phi_{TM} = 2\tan^{-1}\left(-\frac{n_1^2}{n_2^2}\frac{\sqrt{n_1^2 \sin^2\theta_1 - n_2^2}}{n_1 \cos\theta_1}\right)$$

Goos-Hanchen shift (Appendix A)

need to have beam, two incident plane waves
$\beta \pm \Delta\beta$



$$A(z) = \left[e^{j\Delta\beta z} + e^{-j\Delta\beta z}\right] e^{-j\beta z}$$

$$= 2\cos(\Delta\beta z) e^{-j\beta z} \quad @ \ x = 0$$

for small $\Delta\beta$, $\phi(\beta + \Delta\beta) = \phi(\beta) + \frac{d\phi}{d\beta}\Delta\beta$

$$B(z) = 2\cos\Delta\beta(z - 2z_s)\, e^{-j\beta z} \qquad z_s = d\phi/d\beta$$

for TE, $k_0 z_s = \tan\theta / \sqrt{\beta^2 - n_2^2}$ Goos-Hanchen shift

$x = z_s/\tan\theta = 1/\gamma$ attenuation length