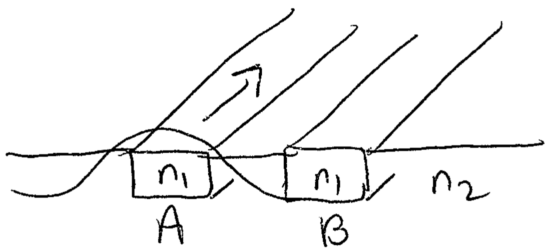
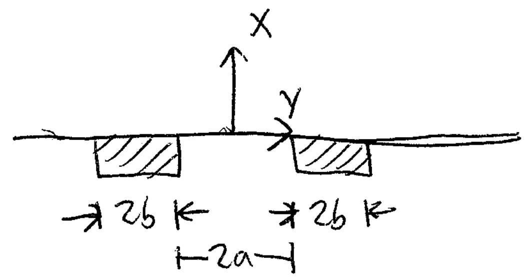
Coupled mode theory - directional coupler





fields of uncoupled w.g.'s (in core)

$$E_A(x,y,z) = A\cos(k_x x + \phi_x)\cos[k_y(y+a+b)]e^{-j\beta z}$$

$$E_B(x,y,z) = B\cos(k_x x + \phi_x)\cos[k_y(y-a-b)]e^{-j\beta z}$$

symmetric along y, $\Rightarrow$ no ϕ_y

polarization perturbation in w.g. B due to evanescent tail of w.g. A, which serves as source term to excite mode B.

the source term for mode B is

$$P_{pert} = \epsilon_0[n_1^2(x,y) - n_2^2]\left[\frac{A}{2}\mathcal{E}_A(y)e^{-j(\beta z - \omega t)} + cc\right]$$

no backward wave, so that

$$-\frac{\partial B}{\partial z}e^{-j(\beta z-\omega t)} + cc = -\frac{j}{2\omega}\frac{\partial^2}{\partial t^2}\int_S \mathcal{E}_B(x,y)P_{pert}\,dS$$

$$= -\frac{j}{2\omega}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{\infty}\epsilon_0[n_1^2(x,y)-n_2^2]\mathcal{E}_B\left[\frac{A}{2}\mathcal{E}_A(y)e^{-j(\beta z-\omega t)} + cc\right]dx\,dy$$

$$= jKAe^{-j(\beta z-\omega t)}$$

for mode B serving as source term for mode A, we get

$$-\frac{\partial A}{\partial z}e^{-j(\beta z-\omega t)} = jKBe^{-j(\beta z-\omega t)}$$ by symmetry

due to strong coupling (i.e. all energy can xfer) must consider coupled equations

$$\frac{\partial A}{\partial z} = -jKB \qquad \frac{\partial B}{\partial z} = -jKA$$

take $\frac{\partial}{\partial z}$ (1) $\quad \frac{\partial^2 A}{\partial z^2} = -jK\frac{\partial B}{\partial z} \Rightarrow \frac{-1}{jK}\frac{\partial^2 A}{\partial z^2} = \frac{\partial B}{\partial z}$

$$+\frac{1}{jK}\frac{\partial^2 A}{\partial z^2} = +jKA \Rightarrow \frac{\partial^2 A}{\partial z^2} = -K^2 A$$

$$A(z) = a\cos(Kz) + b\sin(Kz) \qquad A(0)=1$$

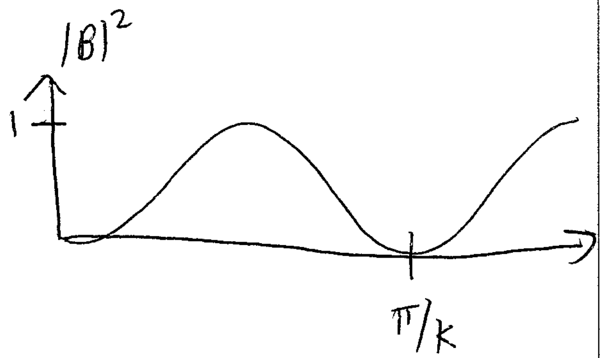
$$\cancel{= a\cos(Kz)} \qquad B(0)=0$$

$$\frac{\partial B}{\partial z} = -jaK\cos(Kz) - jbK\sin(Kz) \qquad \Rightarrow a=1$$

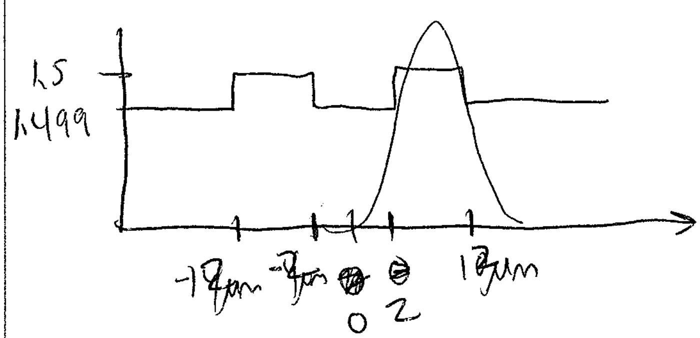
$$B(z) = -ja\sin(Kz) + jb\cos(Kz) \Rightarrow b=0$$

$$A(z) = \cos(Kz)$$

$$B(z) = -j\sin(Kz)$$



example



for isolated w.g. $\lambda = 1\,\mu m$

$\beta = 94227\ cm^{-1}$

$$E_a(x) = \begin{cases} C e^{-2840(x-0.0012)} \\ C\dfrac{\cos[1942(x-.0007)]}{\cos(1942\cdot 0.0005)} \\ C e^{2840(x-0.0002)} \end{cases}$$

using normalization, $C = 433.56$

$P_{pert} = \mathcal{E}_a(x)\,\epsilon_0\,(n_1^2 - n_2^2)\left[\frac{A}{2} e^{-j(\beta z - \omega t)} + cc\right]$ core B

$= 0$ elsewhere

$$K = \frac{\epsilon_0 \omega}{4} \int_{-0.0012}^{-.0002} (1.5^2 - 1.499^2)\, \mathcal{E}_b \mathcal{E}_a \, dx = 3.6217 cm^{-1}$$

spatial coupling length ~ 0.4 cm $\quad z_0 = \frac{\pi}{2K}$

compares w/ z_0 obtained via BPM.