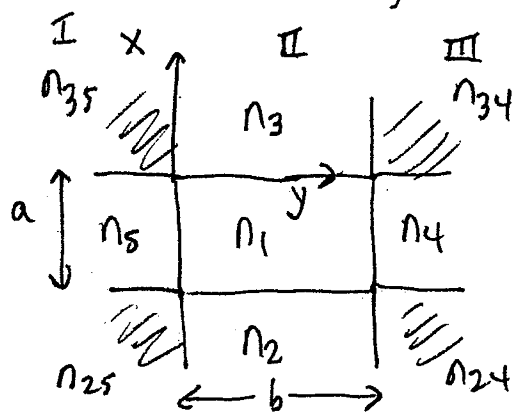
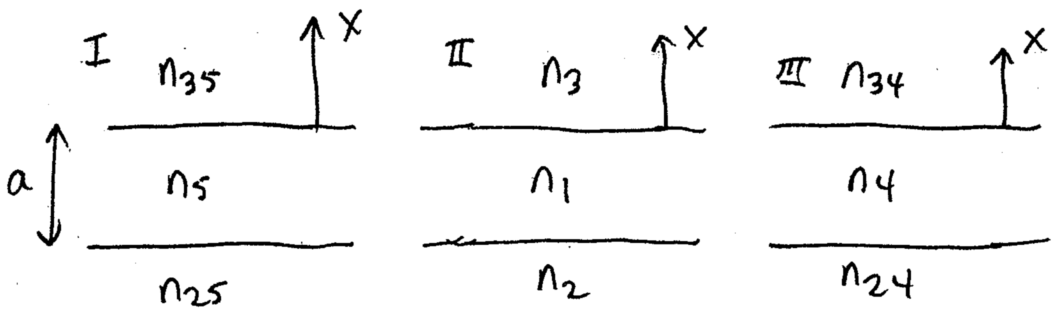
Effective index method

- two orthogonally-oriented waveguides (TE and TM)
- accounts for direct interaction
- converts 2-D into two 1-D problem
- not applicable to square waveguides (aspect ratio>3)
- works well for many ridge waveguides



Step 1

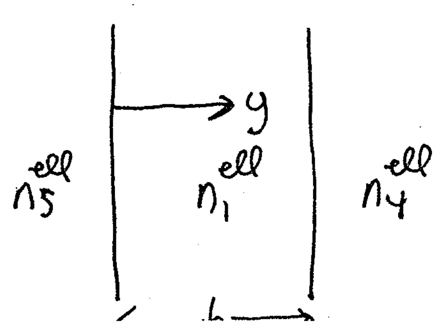
solve for effective indices along horiz. direction
this gives us 3 ~~two~~ "slab" waveguide along x:



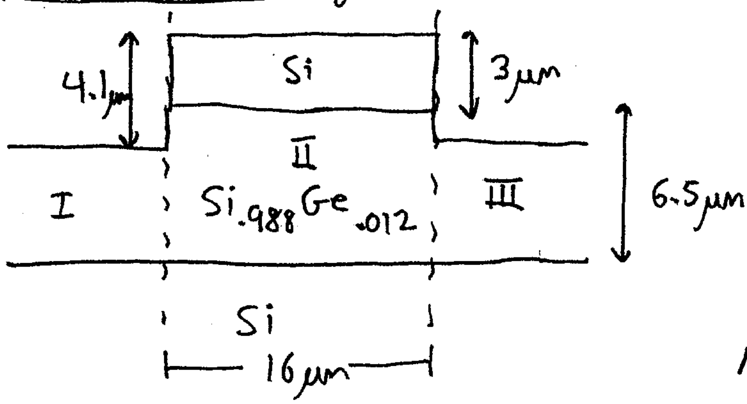
use the appropriate TE or TM slab equations to obtain neff for each of these regions:

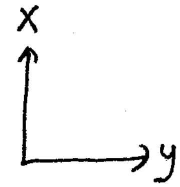
n_5^{eff} $\quad$ n_1^{eff} $\quad$ n_4^{eff}

with these eff. indices, we construct slab wg. along y



and solve for the TM or TE slab mode. the β from this step is the β of the 2-D waveguide. the mode shape is the product of x and y modes ex. SiGe waveguide





$\lambda = 1.32\mu m$

~~Ex~~ E^x

$n_{Si} = 3.5$

$n_{Si_{1-x}Ge_x} = n_{Si} + 0.104x$
$= 3.50125$

region II TM mode

$$\tan K_x h_1/2 = \frac{n_1^2}{n_2^2}\frac{\gamma_x}{K_x} \quad \text{(symmetric wg)}$$

$h_1 = 6.5\mu m \quad n_1 = 3.50125 \quad n_2 = 3.5$

solving, $K_x = 2769.22\ cm^{-1}$

$\beta_x = \sqrt{k_0^2 n_1^2 - K_x^2} = 166636\ cm^{-1} \Rightarrow n_1^{eff} = 3.50077$

$\gamma_x = \sqrt{\beta_x^2 - k_0^2 n_2^2} = \sqrt{k_0^2(n_1^2 - n_2^2) - K_x^2} = 3487\ cm^{-1}$

region I and III TM mode

$$\tan K_f h_2 = K_f\left[\frac{n_f^2}{n_s^2}\gamma_s + \frac{n_f^2}{n_c^2}\gamma_c\right] \Big/ \left[K_f^2 - \frac{n_f^4}{n_c^2 n_s^2}\gamma_c\gamma_s\right]$$

(asymmetric)

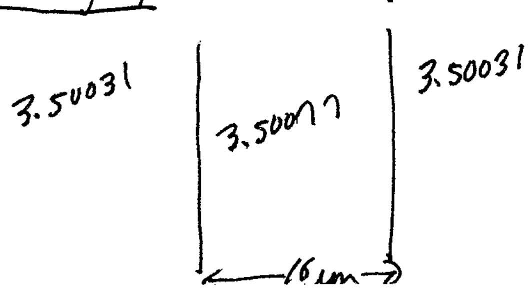
$h_2 = 5.4\mu m,\ n_f = 3.50125,\ n_c = 1,\ n_s = 3.5$

solving, $K = 3866.75\ cm^{-1}$

$\beta = 166614\ cm^{-1} \Rightarrow n_{eff} = 3.50031$

$\gamma_s = \sqrt{k_0^2(n_f^2 - n_s^2) - K^2} \qquad \gamma_c = \sqrt{k_0^2(n_f^2 - n_c^2) - K^2}$

along y (TE mode)



symmetric

$\tan(K_y h_3/2) = \gamma_y / K_y$

$h_3 = 16\mu m \quad$ solving, $K_y = 1323\ cm^{-1}$

$\gamma_y = 2352\ cm^{-1}$

$\boxed{\beta = 166631\ cm^{-1}}$

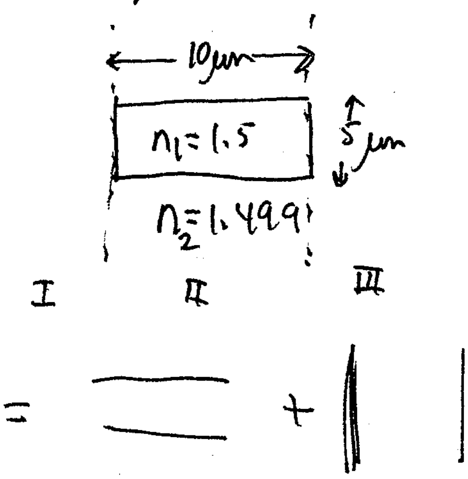
with K_x, γ_x and K_y, γ_y, we can get 2-D mode profile

$$E(x) = \left.\begin{cases} A\hat{x}\,\dfrac{\cos(K_x x)}{\cos(K_x h_1/2)} & |x| < h_1/2 \\ A\hat{x}\,e^{-\gamma_x |x|} & |x| > h_1/2 \end{cases}\right\} \text{symmetric}$$

$$E(y) = \left.\begin{cases} \hat{x}\,\dfrac{\cos(K_y y)}{\cos(K_y h_3/2)} & |y| < h_3/2 \\ \hat{x}\,e^{-\gamma_y |y|} & |y| > h_3/2 \end{cases}\right\} \text{symmetric}$$

(see fig. 5.14)

<u>example 5.1</u> - buried waveguide



no need to solve for I and II ⇒ $n_x^{eff} = n_y^{eff} = 1.499$

E^y

<u>region II</u> symmetric TE

$$\tan(K_x a/2) = \sqrt{k_0^2 n_1^2 - K_x^2}\,/K_x$$

$$n_{eff} = \sqrt{k_0^2 n_1^2 - K_x^2}\,/k_0$$

<u>along y</u> (symmetric TM)

$$\tan(K_y b/2) = \frac{n_{eff}^2}{n_2^2}\sqrt{k_0^2 n_{eff}^2 - K_y^2}\,/K_y$$

$$\beta = \sqrt{k_0^2 n_{eff}^2 - K_y^2} \quad \text{and} \quad b = \frac{\beta^2 - k_0^2 n_2^2}{k_0^2 n_1^2 - k_0^2 n_2^2}$$

(see Fig. 5.16)

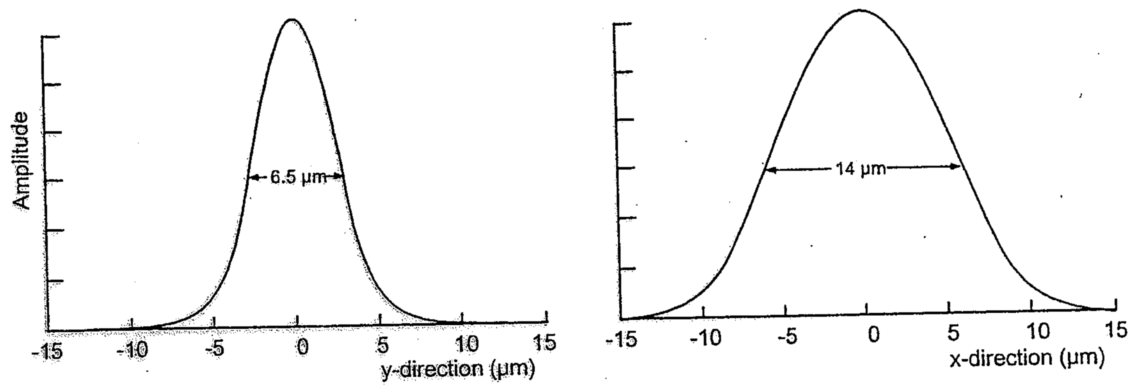


Figure 5.14. The calculated mode intensities derived from the constants given in the text. The calculated dimensions agree well with the experimentally measured values of 8.0 μm, and 13.9 μm.

5. Effective Index Method applied to Ex. 5.1

Let's complete the example we began in the wave analysis and perturbation theory sections by using the effective index method to calculate the normalized propagation coefficients for the waveguide first introduced in Ex. 5.1.

The calculation is relatively straightforward. First, the waveguide is analyzed as if it were a 5μm-thick slab waveguide. Since the electric field is oriented in the y-direction, the mode can be analyzed as a TE mode for the thin dimension. The transverse wavevector, κ_x was found for a range of wavelengths spanning $0.5 \rightarrow 2\mu$m using

$$\tan \kappa_x a/2 = \sqrt{k_0^2 n_1^2 - \kappa_x^2}/\kappa_x \tag{5.38}$$

From this data, an effective index was assigned for each k-vector

$$n_{eff} = \sqrt{k_0^2 n_1^2 - \kappa_x^2}/k_0 \tag{5.39}$$