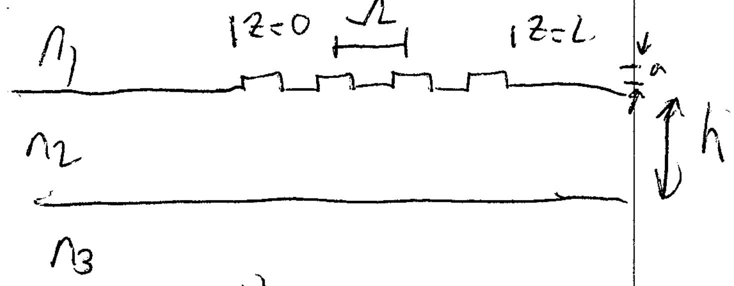
Periodic peturbation

assume lowest-order mode, approx. as gaussian



$$E(x) = \tfrac{1}{2} A^+(z)\, \mathcal{E}_0 e^{-x^2/w^2} e^{j(\beta z - \omega t)} + cc$$

$$P_{pert} = \epsilon_0 \Delta n^2(x,z) \left[\tfrac{1}{2} A^+ \mathcal{E}_0 e^{-x^2/w^2} e^{-j(\beta z - \omega t)} + cc\right]$$

coupling between forward and backward wave

$$\frac{\partial A^-}{\partial z} - \frac{\partial A^+}{\partial z} e^{-2j\beta z} = \frac{j\omega\epsilon_0}{4} A^+ e^{-2j\beta z} \int_0^\infty \Delta n^2(x,z)\, \mathcal{E}_0^2\, e^{-2x^2/w^2} dx$$

assume Δn^2 of the form (fund. component of Δn square wave)

no coupling — coupling w/ $2\pi/\Lambda$

$$\Delta n^2(x,z) = \frac{\Delta n^2(x)}{4} \left[1 + 2\cos(2\pi z/\Lambda) + \cos^2(2\pi z/\Lambda)\right]$$

if we allow $2\pi/\Lambda = 2\beta$, then get backward wave due to phase matching

coupling with $4\pi/\Lambda$

define $\delta = 2\beta - 2\pi/\Lambda$, use middle term

$$2\cos(2\pi z/\Lambda) = e^{j2\pi z/\Lambda} + e^{-j2\pi z/\Lambda}$$
$$= e^{j(2\beta-\delta)z} + e^{-j(2\beta-\delta)z}$$

mult by $e^{-2j\beta z}$ in front of $\int$ and we get

$$\underbrace{e^{-j\delta z}}_{\text{keep}} + e^{-j(4\beta-\delta)z} \leftarrow \text{not phase matched}$$

$$\frac{\partial A^-}{\partial z} = \frac{j\omega\epsilon_0}{4} A^+ e^{-j\delta z} \int_0^a \Delta n^2(x)\, \mathcal{E}_0^2 \left(\tfrac{1}{2}\right) e^{-2x^2/w^2} dx$$

$$= K A^+ e^{-j\delta z} \quad \leftarrow \text{not } \Delta \qquad K = \frac{j\omega\epsilon_0}{4} \Delta n^2 \mathcal{E}_0 \left[\tfrac{1}{2}\left(1 - e^{-2a^2/w^2}\right)\right]$$

similar steps for A^+ ~~using e~~ using A^- as source term.

$$\frac{dA^+}{dz} = K A^- e^{j\delta z}$$

when $\delta = 0$, we get solution,

$$A^-(z) = A^+(0)\, \frac{\sinh[K(z-L)]}{\cosh(KL)}$$

$$A^+(z) = A^+(0)\, \frac{\cosh[K(z-L)]}{\cosh(KL)}$$

see Fig. 10.15

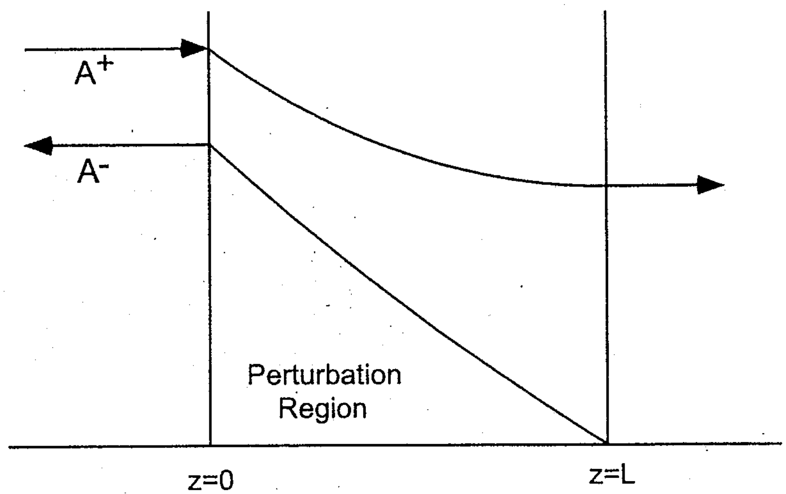


Figure 10.15. The forward and backward amplitudes of an HE_{11} mode in a fiber. Note that amplitudes, not power, are plotted.