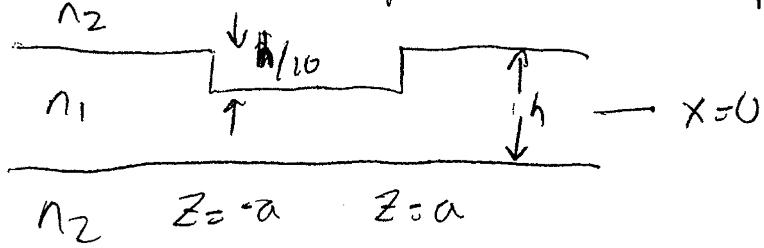
Coupled mode theory - Intermode coupling



launch one mode, couple to another

$$E_a = \frac{A}{2}\mathcal{E}_a(x)e^{-j(\beta_a z - \omega t)} + cc$$

$$E_b = \frac{B}{2}\mathcal{E}_b(x)e^{-j(\beta_b z - \omega t)} + cc$$

for $z < -a$, all power in E_a

$$-\frac{dB^+}{dz}e^{-j(\beta_b z - \omega t)} + \frac{dB^-}{dz}e^{j(\beta_b z + \omega t)} + cc = \frac{-j}{2\omega}\frac{d^2}{dt^2}\int P_{pert}\,\mathcal{E}_b\,dx$$

$$P_{pert} = \epsilon_0(n_2^2 - n_1^2)E_a$$

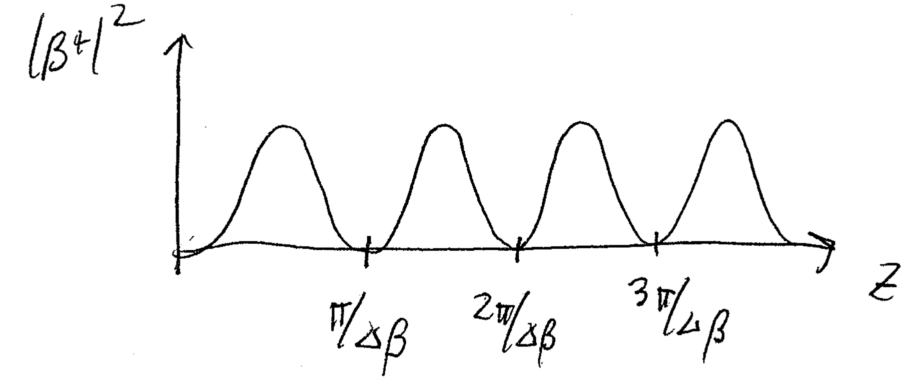
$$= jKA^+e^{-j(\beta_a z - \omega t)}$$

$$-\frac{dB^+}{dz} + \underbrace{\frac{dB^-}{dz}e^{2j\beta_b z}}_{\text{ignore due to fast oscillation}} = jKA^+e^{-j(\beta_a - \beta_b)z}$$

$$K = \frac{\omega\epsilon_0(n_2^2 - n_1^2)}{4}\int_{.4h}^{.5h}\mathcal{E}_a\mathcal{E}_b\,dx$$

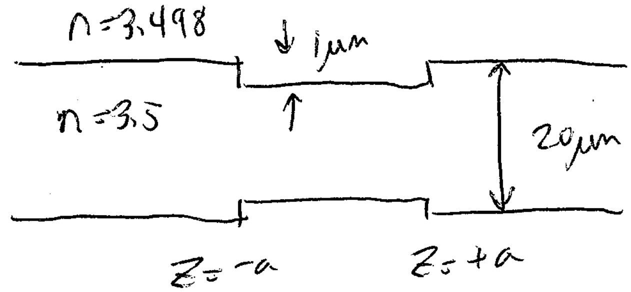
$$B^+(a) = -jKA^+\int_{-a}^{a}e^{-j(\beta_a - \beta_b)z}dz$$

$$= \frac{-2jKA^+}{\beta_a - \beta_b}\sin\left[(\beta_a - \beta_b)a\right]$$



example 10.1

symmetric notch coupling between fundamental TE mode and other TE modes



$\lambda = 1.3\mu m$

step 1 determine modes

$$\tan(\kappa h/2) = \gamma/\kappa \text{ (even) or } -\kappa/\gamma \text{ (odd)}$$

$$\mathcal{E}_y = \begin{cases} C e^{-\gamma(x-h/2)} & x>h/2 \\ C \dfrac{\cos(\kappa x)}{\cos(\kappa h/2)} \text{ or } C \dfrac{\sin(\kappa x)}{\sin(\kappa h/2)} & |x|<h/2 \\ \pm C e^{\gamma(x+h/2)} & x<-h/2 \end{cases}$$

use C to normalize powers (to 1 W/cm)

$$\int \mathcal{E}_n \mathcal{E}_m \, dx = \frac{2\omega\mu}{\beta_n}\delta_{nm}$$

see Table 10.1 and Fig. 10.7

step 2 perturbation (field of TE_{00} mode in notch)

$A_0^+(z) \approx 1$

$$P_{pert} = \epsilon_0 (n_2^2 - n_1^2) C \frac{\cos(\kappa_0 x)}{\cos(\kappa_0 h/2)}\left[\frac{A_0^+}{2} e^{-j(\beta_0 z - \omega t)} + cc\right]$$

$$= -2.647\times10^{-13}\cos(1335.12x)\left[\frac{A_0^+}{2}e^{-j(\beta_0 z-\omega t)} + cc\right]$$

$$K_{0j} = \frac{\omega}{2}\left[\int_{.45h}^{.5h} -2.647\times10^{-13}\cos(1335.12x)\,\mathcal{E}_j\,dx + \int_{-.5h}^{-.45h} -2.647\times10^{-13}\cos(1335.12x)\,\mathcal{E}_j\,dx\right]$$

by symmetry, TE_1 and TE_3 will not couple with TE_0 via a symmetric perturbation. only mode that will couple is TE_2:

$$K_{02} = \omega \int_{.45h}^{.5h} -2.647\times10^{-13} \cos(1335.12x)\, 282.7 \frac{\cos(3949.8x)\,dx}{\cos(3949.8h/2)}$$

$$= -3.912\ cm^{-1}$$

step 3 solve coupled mode eqn.

$$\underbrace{\frac{dA_2^-}{dz} e^{2j\beta_2 z}}_{\approx 0} - \frac{dA_2^+}{dz} + cc = jK_{02} A_0^+ e^{-j(\beta_0-\beta_2)z}$$

$$A_2^+(a) = [2jK_{02}/(\beta_0-\beta_2)]A_0^+ \sin(\beta_0-\beta_2)a$$

$$= j(1.83/40)A_0^+ \sin(\beta_0-\beta_2)a$$

$\beta_0 \neq \beta_2 \Rightarrow$ weak coupling see Fig. 10.8

more general case reciprocity

$$\frac{dA}{dz} = jK_{ab} B e^{-j(\beta_a-\beta_b)z}$$

$$\frac{dB}{dz} = jK_{ba}^* A e^{j(\beta_a-\beta_b)z}$$

and $\frac{d}{dz}(|A|^2+|B|^2)=0$ conservation of power

Table 10.1. Propagation coefficients for the four allowed modes.

Mode Designation	κ	β	Normalization Amplitude
TE_0	1335.12 cm^{-1}	169157 cm^{-1}	99.7 V/cm
TE_1	2658.1 cm^{-1}	169142 cm^{-1}	197.2 V/cm
TE_2	3949.8 cm^{-1}	169117 cm^{-1}	287.7 V/cm
TE_3	5158.45 cm^{-1}	169084 cm^{-1}	353.3 V/cm

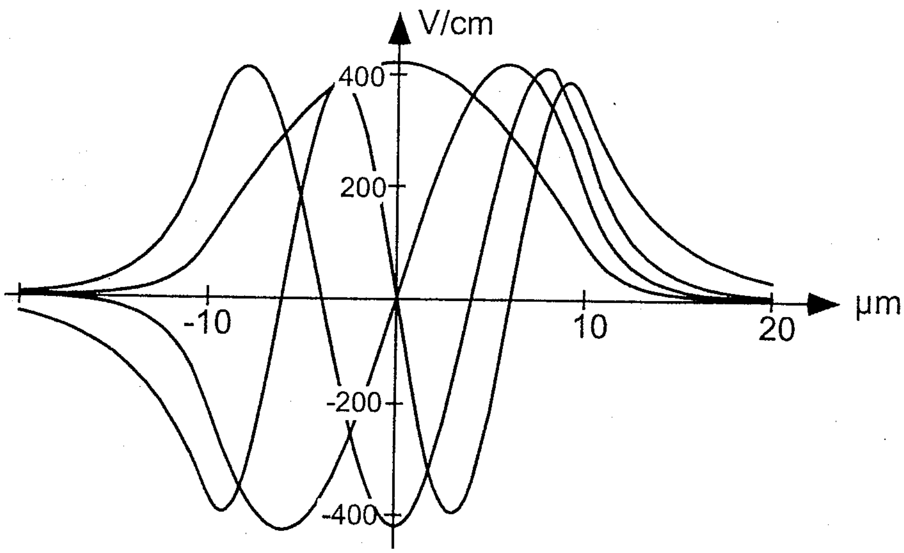


Figure 10.7. The four mode amplitudes are plotted on the same scale. The horizontal axis is in units of cm.

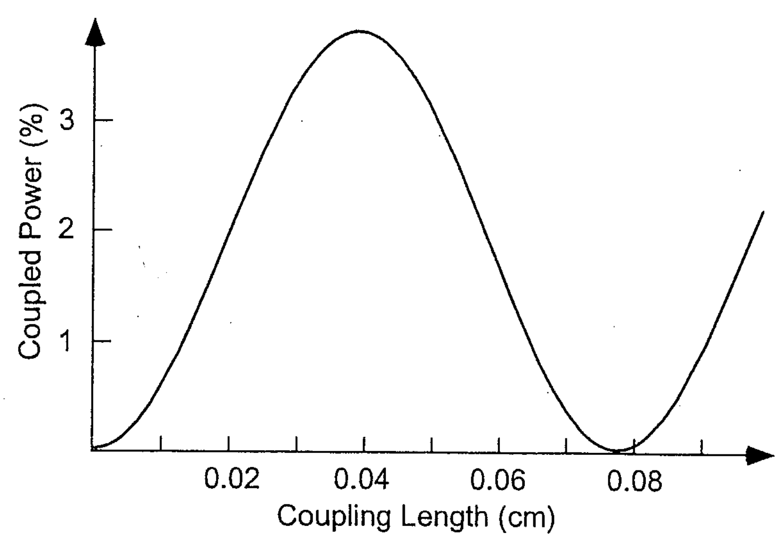


Figure 10.8. The power transfer from mode TE_0 to mode TE_2 as a function of the notch lengtl in Fig.10.6. The coupling varies periodically with notch length.