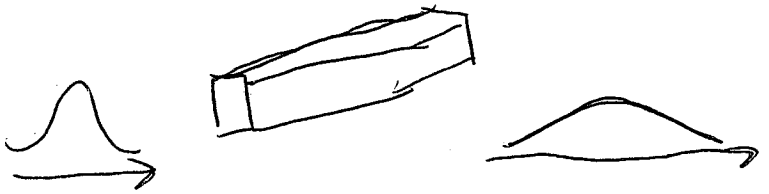
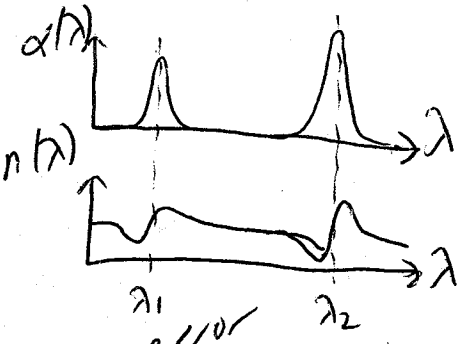
2/18/09

~~Material~~ Dispersion



pulse spread limits data rate

different λ's propagate at different velocities



1. material dispersion $n(\lambda)$
2. waveguide dispersion $n_{eff}(\lambda)$
3. Modal dispersion $n_{eff,min}$

notice error in book

material dispersion

Sellmeir-relations

$$n^2(\lambda) = A + \sum_{j=1}^{M} \frac{B_j \lambda^2}{\lambda^2 - \lambda_j^2}$$

valid away from resonances (i.e. low loss).

or $$n^2(\omega) = A + \sum_{j=1}^{M} \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2}$$

fused silica SiO_2 (show graph)

$A = 1$

$B_1 = 0.6961663$ $\quad \lambda_1 = 0.0684043\,\mu m$

$B_2 = 0.4079426$ $\quad \lambda_2 = 0.1162414\,\mu m$

$B_3 = 0.8974794$ $\quad \lambda_3 = 9.896161\,\mu m$

Al_2O_3 (alumina)

$A = 1$

$B_1 = 1.0238$ $\quad \lambda_1 = 0.0615\,\mu m$

$B_2 = 1.0583$ $\quad \lambda_2 = 0.1107\,\mu m$

$B_3 = 5.2808$ $\quad \lambda_3 = 17.926\,\mu m$

$GeO_2 : SiO_2$ (0.135 : 0.865)

$A = 1$

$B_1 = 0.711040$ $\lambda_1 = 0.064270\ \mu m$

$B_2 = 0.451885$ $\lambda_2 = 0.129408\ \mu m$

$B_3 = 0.704048$ $\lambda_3 = 9.425478\ \mu m$

Sillion - modified sellmeier

$$n^2 = \varepsilon + \frac{A}{\lambda^2} + \frac{B\lambda_1^2}{\lambda^2 - \lambda_1^2}$$

$\varepsilon = 11.6858$ $\lambda_1 = 1.1071\ \mu m$

$A = .939816\ \mu m^2$ valid λ below bandgap $\lambda > \lambda_1$

$B = 8.10461 \times 10^{-3}$

group delay

$$\tau_g = 1/v_g = \frac{dk}{d\omega} \qquad k = \frac{\omega n}{c}$$

$$= \frac{1}{c}\left(n + \omega \frac{dn}{d\omega}\right)$$

normal dispersion

$\frac{dn}{d\lambda} < 0$ $\frac{dn}{d\omega} > 0$

group index

$$n_g = n + \omega \frac{dn}{d\omega} = n - \lambda \frac{dn}{d\lambda}$$

group velocity dispersion

pulse spectral bandwidth $\delta\lambda$

time of flight $\tau = \tau_g \cdot L = \frac{n_g}{c} \cdot L$

spread in time $\delta\tau = \frac{L}{c}[n_g(\lambda_1) - n_g(\lambda_2)]$

$$= \frac{L}{c}\delta n_g = \frac{L}{c}\frac{dn_g}{d\lambda}\delta\lambda$$

what's $\frac{dn_g}{d\lambda}$?

$$\frac{dn_g}{d\lambda} = \frac{d}{d\lambda}\left(n - \lambda \frac{dn}{d\lambda}\right)$$

$$= \frac{dn}{d\lambda} - \frac{dn}{d\lambda} - \lambda \frac{d^2n}{d\lambda^2} = -\lambda \frac{d^2n}{d\lambda^2}$$

$\Delta\tau = \frac{L}{c} D \Delta\lambda$ $\quad D = -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}$ dispersion parameter ps/nm·km

$= L\beta_2 \Delta\omega$ $\quad \beta_2 = \frac{d^2k}{d\omega^2}$ ps^2/km group delay dispersion

$$D = -\frac{2\pi c}{\lambda_0^2}\beta_2$$

$$= -\frac{\lambda_0}{c}\frac{d^2 n_{eff}}{d\lambda^2}$$