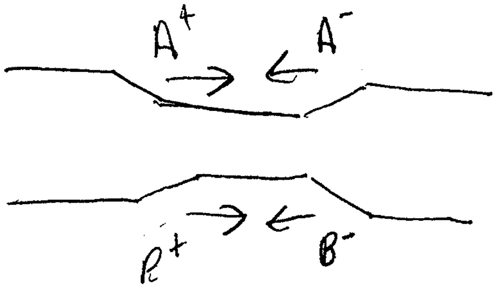
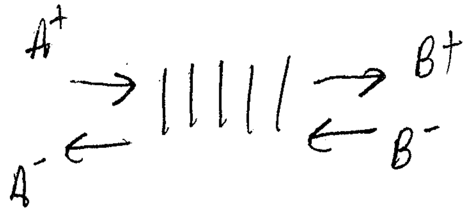
Coupled Mode Theory



2×2 coupler



Bragg grating

scattering from one ~~one~~ mode to another via dielectric perturbation. start off with eigenmodes of unperturbed system, which satisfy

assume TE $\nabla^2 E_y = \epsilon\mu\, \partial^2 E_y/\partial t^2$

$$E_{y_i} = \tfrac{1}{2} A_i \mathcal{E}_{y_i} e^{-j(\beta_i z - \omega t)} + cc$$

A_i = ampl. of mode i

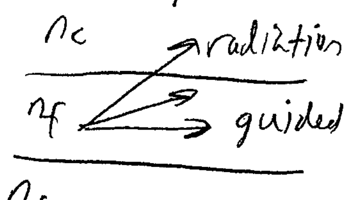
$\mathcal{E}_{y_i}$ = mode field dist.

w/o perturbation, no mode coupling

$$D = \epsilon E + P_{pert}$$

$$\nabla^2 E_y = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2} + \mu \frac{\partial^2 P_{pert}}{\partial t^2}$$

any field dist. can be described as superposition of eigenmodes of system



$$E(x) = \tfrac{1}{2}\sum_i A_i^+(z)\,\mathcal{E}_i(x) e^{-j(\beta_i z - \omega t)}$$

$$+ \tfrac{1}{2}\sum_i A_i^-(z)\,\mathcal{E}_i(x) e^{j(\beta_i z + \omega t)}$$ (forward/backward identical)

$$+ \tfrac{1}{2}\int_{k_0 n_s}^{k_0 n_f} A(\beta, z)\,\mathcal{E}_\beta(x) e^{-j(\beta z - \omega t)}\, d\beta + cc$$ (radiation modes)

want to find coefficients A

plug ~~into~~ $E(x)$ into wave eqn.

ignore radiation modes for now

$$\frac{1}{2}\sum_i \left[\frac{\partial^2 A_i^+}{\partial z^2} - 2j\beta_i \frac{\partial A_i^+}{\partial z}\right] \mathcal{E}_i(x)\, e^{-j(\beta_i z - \omega t)}$$

$$+\frac{1}{2}\sum_i \left[\frac{\partial^2 A_i^-}{\partial z^2} + 2j\beta_i \frac{\partial A_i^-}{\partial z}\right] \mathcal{E}_i(x)\, e^{j(\beta_i z + \omega t)} + cc$$

$$= \mu \frac{\partial^2 P_{pert}}{\partial t^2}$$

(remember $\mathcal{E}_i(x)\, e^{\pm j\beta_i z}$ is a solution to unperturbed w.eq.)

assume slow variation

$$\partial^2 A_i/\partial z^2 \ll 2\beta_i\, \partial A_i/\partial z$$

$$\frac{1}{2}\sum_i \left[-2j\beta_i \frac{\partial A^+}{\partial z}\right] \mathcal{E}_i(x)\, e^{-j(\beta_i z - \omega t)}$$

$$+\frac{1}{2}\sum_i \left[2j\beta_i \frac{\partial A^-}{\partial z}\right] \mathcal{E}_i(x)\, e^{j(\beta_i z + \omega t)} + cc = \mu \frac{\partial^2 P_{pert}}{\partial t^2}$$

simplify using mode orthogonality

$$\frac{1}{2}\int \mathcal{E}_i(x) \times \mathcal{H}_j(x)\, dx = \delta_{ij} \qquad i,j = \text{mode indices}$$

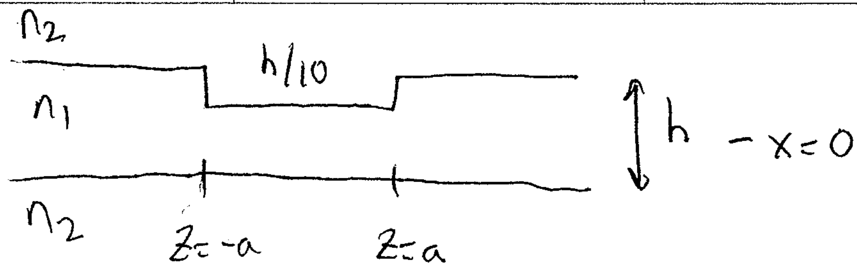
for TE mode, H_x can be written in terms of E_y

$$\int \mathcal{E}_i(x)\, \mathcal{E}_j(x)\, dx = \frac{2\omega\mu}{\beta_j}\delta_{ij}$$

multiply by $\mathcal{E}_j$ and integrate

$$\boxed{\frac{\partial A_j^-}{\partial z} e^{j(\beta_j z + \omega t)} - \frac{\partial A_j^+}{\partial z} e^{-j(\beta_j z - \omega t)} + cc = \frac{-j}{2\omega}\frac{\partial^2}{\partial t^2}\int P_{pert}(x)\cdot \mathcal{E}_j(x)\, dx}$$

ex.



unperturbed modes $\quad \mathcal{E}_y = C e^{-\gamma(x-h/2)} \quad x \geq h/2$

$C \cos Kx / \cos(Kh/2) \quad |x| < h/2$

$C e^{\gamma(x+h/2)} \quad x < h/2$

perturbation

$$\Delta\epsilon = \epsilon_0 (n_2^2 - n_1^2)$$

$$P_{pert}(x) = \Delta\epsilon E_1$$

$$= \epsilon_0 (n_2^2 - n_1^2) \left[\tfrac{1}{2} A^+ \mathcal{E}(x) e^{-j(\beta z - \omega t)} + cc \right]$$

for $.4h < x < .5h$, $-a < z < a$

$= 0$ elsewhere

$$\frac{\partial A^-}{\partial z} e^{j(\beta z + \omega t)} - \frac{\partial A^+}{\partial z} e^{-j(\beta z - \omega t)} = \frac{-j}{2\omega} \int_{.4h}^{.5h} P_{pert} \mathcal{E}_y(x) dx$$

$$= -\frac{j}{2\omega} \frac{\partial^2}{\partial t^2} \int \epsilon_0 (n_2^2 - n_1^2) |C|^2 \left(\frac{\cos Kx}{\cos Kh/2} \right)^2 \cdot \tfrac{1}{2} A^+ e^{-j(\beta z - \omega t)} dx$$

$$= \frac{j\omega \epsilon_0 (n_2^2 - n_1^2)}{2} \left[\tfrac{1}{2} A^+ e^{-j(\beta z - \omega t)} \right] \int |C|^2 \left(\frac{\cos Kx}{\cos Kh/2} \right)^2 dx$$

$$= j K A^+ e^{-j(\beta z - \omega t)}$$

$$K = \text{coupling constant} = \frac{\omega \epsilon_0 (n_2^2 - n_1^2)}{4} \int |C|^2 \left(\frac{\cos Kx}{\cos Kh/2} \right)^2 dx \quad -a < z < a$$

$= 0$ elsewhere

equation of motion

$$\frac{\partial A^-}{\partial z} e^{2j\beta z} - \frac{\partial A^+}{\partial z} = jKA^+$$

integrate from $-a < z < a$

if $2a \gg 1/2\beta$, then 1st term averages to zero and we have

$$-\frac{\partial A^+}{\partial z} = jKA^+ \Rightarrow A^+(z) = A^+(-a)\, e^{-jKz}$$

perturbation alters phase of A^+

so forward wave $E(z) = \dfrac{A^+(-a)}{2} e^{-j(\beta+K)z} \quad -a < z < a$

to deal w/ A^-, we assume $dA^+/dz = 0$

so that $\dfrac{\partial A^-}{\partial z} = jKA^+ e^{-2j\beta z} \quad -a < z < a$

$$\Rightarrow A^-(-a) = -j\frac{KA^+}{\beta} \sin[2\beta a]$$

$A^-(a) = 0$ (backward wave)

coupling max. when $2\beta a = \pi/2 \Rightarrow a = \pi/4\beta$

$\beta = 2\pi n_{eff}/\lambda \qquad a = \lambda/8 n_{eff} \qquad 2a = \lambda/4 n_{eff}$

$$2\beta a = (q+1/2)\pi \Rightarrow a = \frac{(q+1/2)\pi}{2\beta}$$