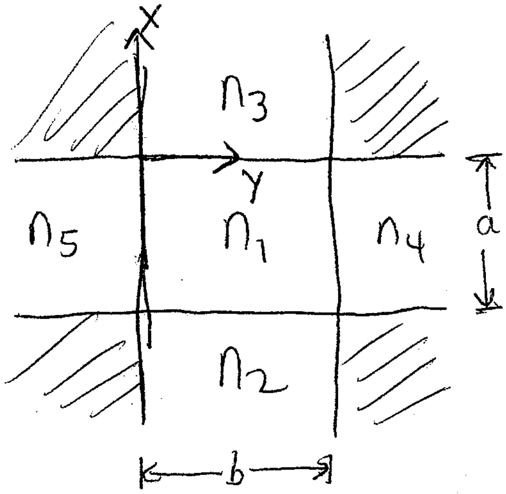
Rectangular waveguides (see figure 5.1)

HW 5.3, 5.4, 5.8, 5.9

3 types: surface waveguide, buried, ridge



four corner regions couple x and y solution

can ignore for large V number, or strong mode confinement

modes neither perfect TE nor TM

for small Δn, fields along x or y axis predominantly, so designate E^y_{nm} or E^x_{nm}

n and m designate # of maxima along x and y

first assume $E(x,y,z) = E(x,y)e^{-j\beta z}$

so that
$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \left[k_0^2 n^2(x,y) - \beta^2\right] E = 0$$

due to rectangular symmetry, under weak guidance, x and y polarizations don't mix, so assume $E(x,y) = X(x)\,Y(y)$

$$\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + k_0^2 n^2(x,y) - \beta^2 = 0 \qquad \ddot{X} = \frac{\partial^2 X}{\partial x^2}$$

$$\text{or} \quad \frac{\ddot{X}}{X} = -k_0^2 n^2(x,y) + \beta^2 - \frac{\ddot{Y}}{Y} \equiv -K_x^2 \qquad K_x^2 \text{ is sep. const.}$$

$$\Rightarrow \frac{\ddot{Y}}{Y} = -k_0^2 n^2(x,y) + \beta^2 + K_x^2 = -K_y^2$$

$$\beta^2 = k_0^2 n^2(x,y) - K_x^2 - K_y^2$$

$e^{-\gamma_3 x}$ $e^{\gamma_5 y}$	$\cos(K_y y+\phi_y)$ $e^{-\gamma_3 x}$ 3	$e^{-\gamma_3 x}$ $e^{-\gamma_4 (y-b)}$
$\cos(K_x x+\phi_x)$ $e^{\gamma_5 y}$ 5	$\cos(K_x x+\phi_x)$ $\cos(K_y y+\phi_y)$ 1	$\cos(K_x x+\phi_x)$ $e^{-\gamma_4 (y-b)}$ 4
$e^{\gamma_2 (x-a)}$ $e^{\gamma_5 y}$	$\cos(K_y y+\phi_y)$ $e^{\gamma_2 (x-a)}$ 2	$e^{-\gamma_4 (y-b)}$ $e^{\gamma_2 (x-a)}$

$\gamma_2 = \sqrt{k_0^2(n_1^2-n_2^2)-K_x^2}$

$\gamma_4 = \sqrt{k_0^2(n_1^2-n_4^2)-K_y^2}$

$\gamma_5 = \sqrt{k_0^2(n_1^2-n_5^2)-K_y^2}$

in the core, region 1: $X(x) = A\cos(K_x x+\phi_x)$

$Y(y) = B\cos(K_y y+\phi_y)$

in region 3, no variation along y going from 1 to 3, so

$Y(y) = B\cos(K_y y+\phi_y)$

and $\frac{\ddot{Y}}{Y} = -K_y^2$

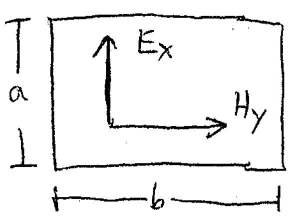
so that $\frac{\ddot{X}}{X} + k_0^2 n^2(x,y) - \beta^2 - K_y^2 = 0 \qquad n^2(x,y) = n_3^2$

or $\frac{\ddot{X}}{X} = (k_0^2 n_1^2 - k_0^2 n_3^2 - K_x^2)$ using $\beta^2 = k_0^2 n_1^2 - K_x^2 - K_y^2$

which has solution $X_3 = e^{-\gamma_3 x}$

$\gamma_3 = \sqrt{k_0^2(n_1^2-n_3^2)-K_x^2}$

See fig. 5.5



(E^x_{nm})

matching fields along boundary

continuity of E_x @ $y=0, y=b$

continuity of H_y @ $x=0, x=a$

this arrangement looks like TE along y and TM along x

$\tan(K_y b) = \frac{K_y(\gamma_4+\gamma_5)}{K_y^2 - \gamma_4\gamma_5}$ identical to TE, slab of thick (note: we don't know β yet)

$$\tan(k_x a) = n_1^2 \frac{k_x(n_2^2\gamma_3 + n_3^2\gamma_2)}{n_2^2 n_3^2 k_x^2 - n_1^2\gamma_2\gamma_3}$$ TM mode in slab of thickness a

knowing k_y and k_x, we can get β

and $\tan\phi_x = -\frac{n_3^2}{n_1^2}\frac{k_x}{\gamma_3}$ $\tan\phi_y = -\frac{\gamma_5}{k_y}$

E^y_{nm}

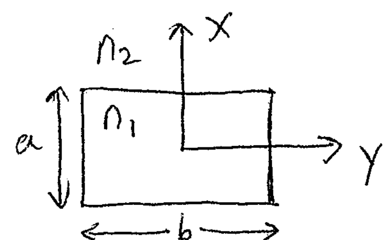
$$\tan(k_x a) = \frac{k_x(\gamma_2+\gamma_3)}{k_x^2 - \gamma_2\gamma_3}$$ TE

$$\tan(k_y b) = n_1^2 \frac{k_y(n_5^2\gamma_4 + n_4^2\gamma_5)}{n_4^2 n_5^2 k_y^2 - n_1^2\gamma_4\gamma_5}$$ TM

$\tan\phi_x = \frac{\gamma_3}{k_x}$ $\tan\phi_y = \frac{n_5^2}{n_1^2}\frac{k_y}{\gamma_5}$

error results from the fact that x and y solutions are uncoupled, and treated as two separate (independent) slab waveguide problems.

Buried waveguide - two symmetric waveguides, oriented ⊥



$n_1 = 1.5$ $n_2 = 1.499$
$a = 5\mu m$ $b = 10\mu m$
$\lambda = .5\mu m$ to $2\mu m$

symmetric modes

$$X(x) = A\frac{\cos(k_x x)}{\cos(k_x a/2)} \qquad |x| < a/2$$

$$A e^{-\gamma_x(|x| - a/2)} \qquad |x| > a/2$$

$$Y(y) = B\frac{\cos(k_y y)}{\cos(k_y b/2)} \qquad |y| < b/2$$

$$B e^{-\gamma_y(|y| - b/2)} \qquad |y| > b/2$$

assume E^y_{mn} mode

$$\tan(\kappa_x a/2) = \frac{\gamma_x}{\kappa_x} = \frac{\sqrt{k_0^2(n_1^2-n_2^2)-\kappa_x^2}}{\kappa_x} \quad TE$$

$$\tan(\kappa_y b/2) = \frac{n_1^2}{n_2^2}\frac{\gamma_y}{\kappa_y} = \frac{n_1^2}{n_2^2}\frac{\sqrt{k_0^2(n_1^2-n_2^2)-\kappa_y^2}}{\kappa_y}$$

$$\beta^2 = k_0^2 n_1^2 - \kappa_y^2 - \kappa_x^2$$

(see fig. 5.8) analysis valid for $V>2$.

for $V<2$, cannot neglect fields in corner regions.

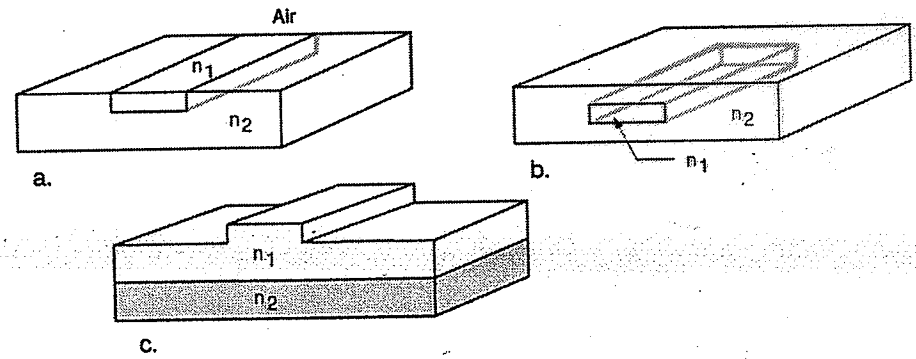


Figure 5.1. Three possible configurations for rectangular waveguides. In a) the cover index is air. In b) the guiding layer is completely surrounded by a cladding layer. In c) lateral confinement is established by the dielectric ridge on top of the substrate.

to control this information varies with the device. Power splitters and interferometers might use Y-junctions, while filters might rely on evanescent coupling between adjacent waveguides. Design and analysis of these problems requires knowing the exact mode structure of the field in order that coupling can be accurately predicted. We will see that unlike the planar slab waveguide or the circularly symmetric fiber waveguide, it is generally impossible to find exact analytical solutions to these structures. Most work is now done using numerical simulations, which are described in a following chapter.

2. Wave Equation Analysis of a Rectangular Waveguide

Fig. 5.1 shows three types of rectangular waveguide that can be employed in an integrated optical circuit. They illustrate the surface waveguide, the buried waveguide, and the ridge waveguide. These geometries are relatively simple to create using standard lithographic and overgrowth techniques. As usual, the index of the rectangular region must be slightly larger than the surrounding medium for the structure to guide light. Our goal is to determine the mode structure of these waveguides. To begin the analysis we will develop a wave equation expression that is accurate well above cutoff.

A cross-section of a generalized embedded waveguide is shown in Fig. 5.2. There are nine distinct dielectric regions in this structure. Analysis is difficult because it is impossible to simultaneously satisfy all the boundary conditions in this structure.

The difficulty in analyzing this structure originates in the four shaded regions. These regions act as the coupling zones for the x and y solutions of the field. Well above cut-off, the electromagnetic mode is tightly confined within the core, and the amount of energy in the corner regions is negligible, so the wave equation can be solved using standard separation of variables. Near cutoff, however, the mode will have a significant amount of power in the corner

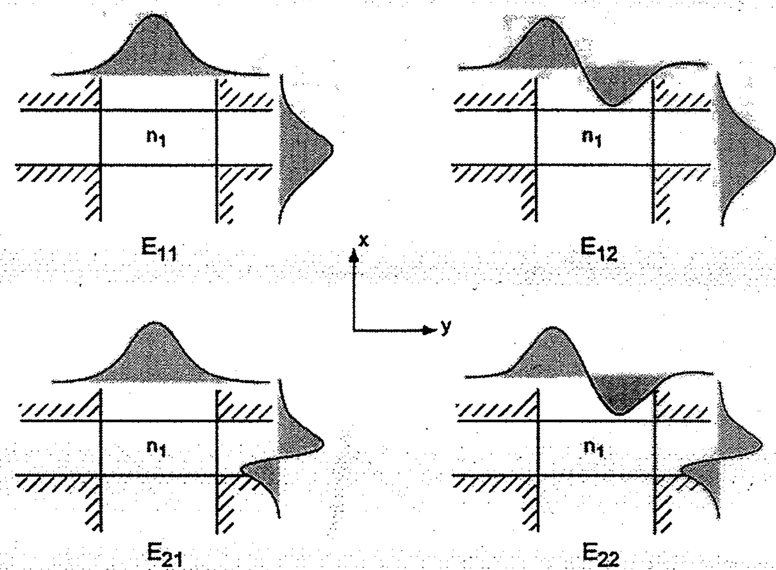


Figure 5.5. The transverse scalar field distributions for the x and y directions.

solutions between the various regions. Since there are many fields, and many interfaces, matching all the boundary conditions is a complicated and tedious process (see for example [2]). Consider the situation where an E^x mode is being guided. Fig. 5.6 shows a cross-section of the waveguide, along with the two transverse components of the field in the core.

In this waveguide, we must insure the continuity of the tangential electric field at the $y = 0$ and $y = b$ planes, and continuity of the tangential magnetic field at the $x = 0$ and $x = -a$ planes. The other boundary conditions (continuity of D_{norm} and B_{norm}) are almost automatically satisfied in the weakly guiding approximation, at least they are close enough to be insignificant. From the boundary condition point-of-view, the E^x field looks like a TE mode in a slab waveguide of thickness b, and a TM mode in a slab waveguide of thickness a. After lengthy calculation [1], the characteristic equation for κ_y can be shown

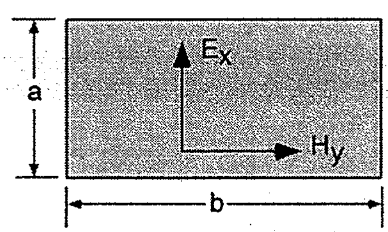


Figure 5.6. The E^x field in the core will have the electric field polarized along the x direction, and the magnetic field polarized along the y direction.

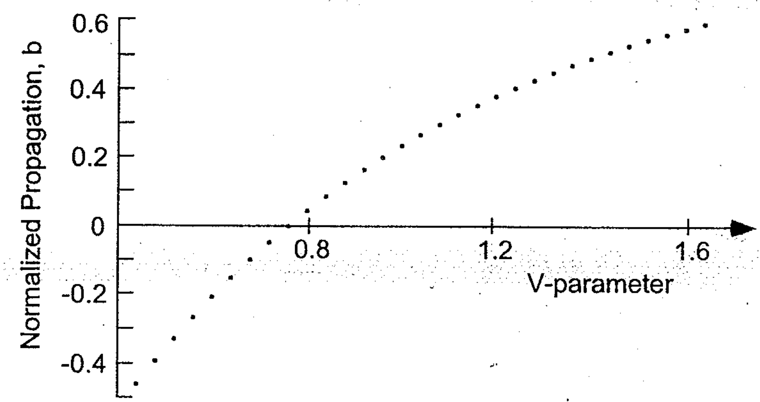


Figure 5.8. The normalized propagation coefficient for the waveguide. Note that b is less than zero for certain ranges of the normalized frequency. This is unphysical, and these modes are therefore below cut-off.

not physical; the only interpretation of these points is that they represent modes that are beyond cut-off. Unfortunately, in a symmetric waveguide we expect that there will always be at least one allowed mode, in contrast to what the numeric results are telling us. It turns out that the calculated solution is increasingly inaccurate in the region of normalized frequency $V < 2$. This is an example of where we must be careful about neglecting the corner dielectric regions. The wave equation solution is valid, however, for values of $V > 2$, as we shall show in the next section.

2.4 Solutions Near Cut-Off

The analysis presented above is an approximation based on neglect of certain regions of the waveguide. So long as the mode is well above cut-off, the solutions and expressions for the eigenvalues will be nearly indistinguishable from the exact value. For modes where $V < 2$, we can expect the exact solution to deviate from the calculated value because there will be non-negligible fields in the corner regions. To reduce this error, one must resort to numerical techniques, or to perturbation methods.

The results of Example 5.1 are disturbing: we expect that for a symmetric waveguide, there will always be at least one guided mode. In fact, this is true. So the calculations are in error. The next section deals with a first order correction to this problem.

3. Perturbation Approach to Correcting β

The major problem with the analytical approach is that it relies on the mode being tightly coupled to the core, so that relatively little field exists in the four corner regions. The source of error that arose in the last example came from

$\exp(-\gamma_3 x)$ $\exp(\gamma_5 y)$	$\cos(\kappa_y y + \Phi_y)$ $\exp(-\gamma_3 x)$ 3	$\exp(-\gamma_3 x)$ $\exp(-\gamma_4 (y-b))$
$\cos(\kappa_x x + \Phi_x)$ $\exp(\gamma_5 y)$ 5	$\cos(\kappa_x x + \Phi_x)$ $\cos(\kappa_y y + \Phi_y)$ 1	$\cos(\kappa_x x + \Phi_x)$ $\exp(-\gamma_4 (y-b))$ 4
$\exp(\gamma_2 (x-a))$ $\exp(\gamma_5 y)$	$\cos(\kappa_y y + \Phi_y)$ $\exp(\gamma_2 (x-a))$ 2	$\exp(-\gamma_4 (y-b))$ $\exp(\gamma_2 (x-a))$

Figure 5.4. The rectangular waveguide can be described as nine separate regions, each with its own electromagnetic field description. (For simplicity, amplitudes are not matched across boundaries in these expressions.)

where

$$\gamma_3 = \sqrt{k_0^2(n_1^2 - n_3^2) - \kappa_x^2} \tag{5.11}$$

So the total field in region 3 can be described as

$$E(x,y) = C\cos(\kappa_y y + \phi_y)e^{-\gamma_3 x} \quad \text{for } x > 0,\ 0 < y < b \tag{5.12}$$

Through a similar set of solutions we can find the scalar solutions to the x- and y-components of the field in regions 2, 4, and 5

$$\begin{aligned} X_2(x) &= e^{\gamma_2(x+a)} \quad \text{for } x < b \\ Y_4(y) &= e^{-\gamma_4(y-b)} \quad \text{for } y > a \\ Y_5(y) &= e^{\gamma_5 y} \quad \text{for } y < 0 \end{aligned} \tag{5.13}$$

where the exponential decay constants are given by

$$\begin{aligned} \gamma_2 &= \sqrt{k_0^2(n_1^2 - n_2^2) - \kappa_x^2} \\ \gamma_4 &= \sqrt{k_0^2(n_1^2 - n_4^2) - \kappa_y^2} \\ \gamma_5 &= \sqrt{k_0^2(n_1^2 - n_5^2) - \kappa_y^2)} \end{aligned} \tag{5.14}$$

These fields are summarized in Fig. 5.4, where the appropriate product of $X(x)$ and $Y(y)$ solutions are listed in each region, and the transverse solutions for several modes are plotted in Fig. 8 5.

2.3 Solution to the Boundary Value Problems

To complete the solution, we must determine the specific values for κ_x, κ_y and β. This is done by applying the boundary conditions that connect the