Slab waveguide eigenvalues

$$E_y(x) = \begin{cases} Ae^{-\gamma_c x} & x > 0 \\ B\cos(k_f x) + C\sin(k_f x) & -h < x < 0 \\ De^{\gamma_s (x+h)} & x < -h \end{cases}$$

need to find unknown coefficients

use continuity of E and H fields, tangential components

to find H field, use

$$\nabla \times \vec{E} = -j\mu_0 \omega \vec{H} \Rightarrow H_z = \frac{j}{\mu_0 \omega} \frac{\partial E_y}{\partial x}$$

in other words, $\frac{\partial E_y}{\partial x}$ continuous across interface

$x=0$: $Ae^{-\gamma_c \cdot 0} = B\cos(k_f \cdot 0) + C\sin(k_f \cdot 0) \Rightarrow A = B$

$$-\gamma_c A e^{-\gamma_c \cdot 0} = -k_f B \sin(k_f \cdot 0) + k_f C \cos(k_f \cdot 0)$$

$$\Rightarrow C = -\frac{\gamma_c}{k_f} A$$

$x=-h$:

$$A\left[\cos(k_f h) + \frac{\gamma_c}{k_f}\sin(k_f h)\right] = De^{\gamma_s (x-h)}\Big|_{x=-h} = D$$

no need to go further. we now have

$$E_y(x) = \begin{cases} Ae^{-\gamma_c x} & x > 0 \\ A\left[\cos(k_f x) - \frac{\gamma_c}{k_f}\sin(k_f x)\right] & -h < x < 0 \\ A\left[\cos(k_f h) + \frac{\gamma_c}{k_f}\sin(k_f h)\right] e^{\gamma_s (x+h)} & x < -h \end{cases}$$

now, need to know β to determine γ_s, γ_c and k_f

use continuity of $\frac{\partial E_x}{\partial x}$ @ $x=-h$

$$A\left[k_f \sin(k_f h) - \gamma_c \cos(k_f h)\right] = A\left[\cos(k_f h) + \frac{\gamma_c}{k_f}\sin(k_f h)\right]\gamma_s$$

$$\Rightarrow \tan(k_f h) = \frac{\gamma_c + \gamma_s}{k_f\left[1 - \frac{\gamma_c \gamma_s}{k_f^2}\right]}$$ transcendental

yield eigenvalues β_{TE}

TM: $$\tan(k_f h) = \frac{k_f\left[\frac{n_f^2}{n_s^2}\gamma_s + \frac{n_f^2}{n_c^2}\gamma_c\right]}{k_f^2 - \frac{n_f^4}{n_c^2 n_s^2}\gamma_c\gamma_s}$$

can also define mode effective index

$$n_{eff} = \beta / k_0$$

<u>example</u>

$n_c = 1.40$

$n_f = 1.50$ ↕ $5\mu m$

$n_s = 1.45$

$\lambda_0 = 1\mu m$

TE $n_{eff} = 1.4974$
1.4898
1.4773
1.4603

TM $n_{eff} = 1.4974$
1.4896
1.4768
1.4597

HW #3.1, 3.4, 3.8, 3.9, 3.22
write computer code, Matlab.