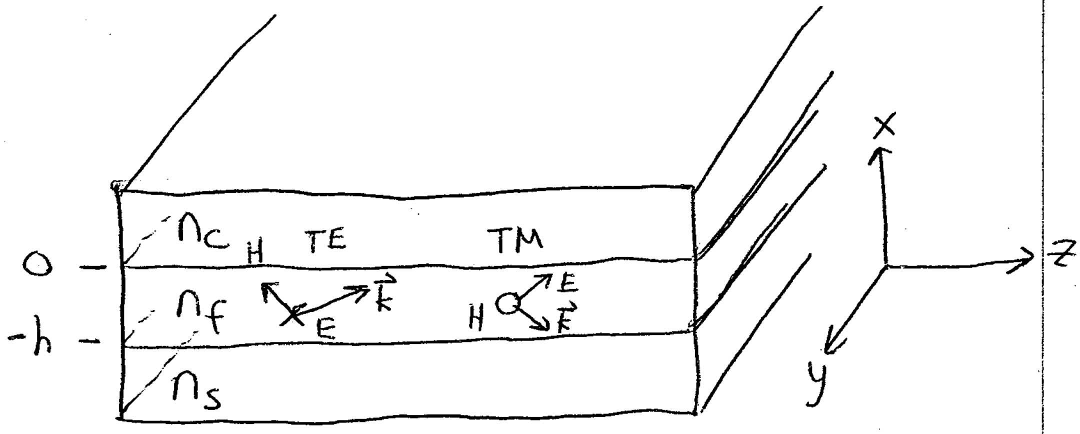
Planar slab waveguide



assume propagation along z

TE – electric field along y axis

TM – magnetic field along y axis

we'll consider TE case in detail.

$$\nabla^2 E_y + k_0^2 n_i^2 E_y = 0 \qquad n_i = \begin{cases} n_c & x>0 \\ n_f & -h<x<0 \\ n_s & x<-h \end{cases}$$

separate wave eqns in each of three regions, solve then connect through boundary conditions

trial solution $\quad E_y(x,z) = E_y(x)\, e^{-j\beta z}$

β = unknown propagation constant

$$\nabla^2 E_y = -\beta^2 E_y(x) + \frac{\partial^2 E_y}{\partial x^2} e^{-j\beta z} = -k_0^2 n_i^2 E_y e^{-j\beta z}$$

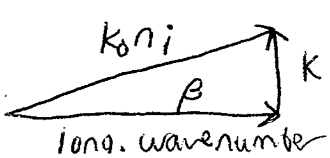
$$\frac{\partial^2 E_y}{\partial x^2} + (k_0^2 n_i^2 - \beta^2) E_y = 0$$

2 cases:

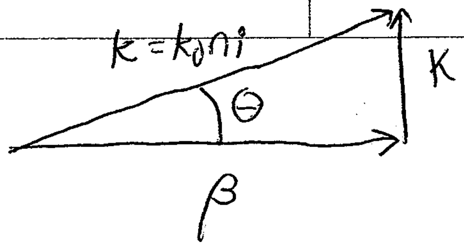
$\beta > k_0 n_i \qquad E_y(x) = E_0 e^{\pm\sqrt{\beta^2 - k_0^2 n_i^2}\, x} \qquad$ $x>0$, choose '−'; $x<-h$, choose '+'

$\beta < k_0 n_i \qquad E_y(x) = E_0 e^{\pm j\sqrt{k_0^2 n_i^2 - \beta^2}\, x}$

define $\gamma = \sqrt{\beta^2 - k_0^2 n_i^2}$ attenuation const.



$K = \sqrt{k_0^2 n_i^2 - \beta^2}$ transverse wavenumber



$\beta = k\cos\theta$

assume $n_c < n_s < n_f$

for $\beta < k_0 n_c$, all three solutions are oscillatory and the wave is not guided – radiation mode

for $k_0 n_c < \beta < k_0 n_s$, oscillatory in core and substrate, decaying (i.e. evanescent) in cover region, known as substrate mode.

for $k_0 n_s < \beta < k_0 n_f$ guided mode. oscillatory in core, exponential decay in cover and substrate. in the ray picture, ray undegoes TIR at each interface