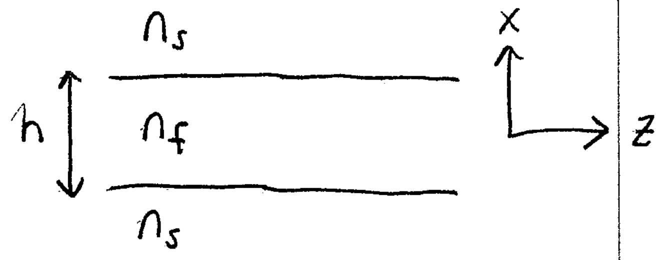
Symmetric waveguide



TE

$$E_y(x) = \begin{cases} A e^{-\gamma(x-h/2)} \\ A\cos(Kx)/\cos(Kh/2) \quad \text{or} \quad A\sin(Kx)/\sin(Kh/2) \\ \pm A e^{\gamma(x+h/2)} \end{cases}$$

$\frac{d}{dz}\sin Kh$?

$$\tan Kh/2 = \begin{cases} \gamma/K & \text{even (symmetric)} \\ -K/\gamma & \text{odd (antisymmetric)} \end{cases}$$

~~for~~ TM

$$\tan Kh/2 = \begin{cases} (n_f/n_s)^2\gamma/K & \text{even} \\ -(n_f/n_s)^2 K/\gamma & \text{odd} \end{cases}$$

ex

$n_f = 1.49 \quad n_s = 1.485 \quad \lambda = .8\mu m \quad h = 3\mu m, 15\mu m$

$\gamma = \sqrt{9.176\times10^{7} - K^2} \quad \beta = \sqrt{1.3694\times10^{10} - K^2}$

(see Figs. 3.10 + 3.11) always 1 mode (TE) and 1 TM

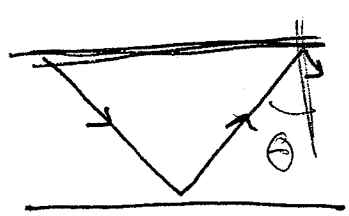
Mode shape

even TE $\quad E_y = A\cos(Kx)e^{-j\beta z}$

$$= \frac{A}{2}\left[e^{j(Kx-\beta z)} + e^{-j(Kx-\beta z)}\right]$$

superposition of 2 plane waves

constructive + destructive interference gives mode shape (see figure 3.12)



for stable profile, total round trip phase a multiple of 2π

$$2kn_f h\cos\theta - 2\Phi_c - 2\Phi_s = 2\pi\nu$$

same as eigenvalue equation $\quad 2\Phi = 2\tan^{-1}\left[-\frac{\sqrt{n_1^2\sin^2\theta_1 - n_2^2}}{n_1\cos\theta_1}\right]$

number of modes

$\beta_{\text{lowest order}} \sim k_0 n_f$

$\beta_{\text{highest order}} \sim k_0 n_f \cos\theta_{cr} \sim k_0 n_s$

the rest fall between these values

$$\tan(Kh) = \frac{\gamma_c + \gamma_s}{K(1 - \gamma_c\gamma_s/K^2)} \quad \text{(see fig. 3.13)}$$

range of β's defines K_{max} via

$$k_0 n_s \approx \sqrt{(k_0 n_f)^2 - K_{max}^2}$$

every increment of hK_{max} by π gives another mode

$$\#\text{modes} = m = \text{Int}[h K_{max}/\pi]$$

$$\approx \text{Int}\left[h k_0 \sqrt{(n_f^2 - n_s^2)}/\pi\right]$$

normalized frequency

$$V = h k_0 \sqrt{(n_f^2 - n_s^2)} \Rightarrow m \approx V/\pi$$

$b = (n_{eff}^2 - n_s^2)/(n_f^2 - n_s^2)$ norm. index

$a = (n_s^2 - n_c^2)/(n_f^2 - n_s^2)$ asymmetry TE

$n_{eff} = \beta/k_0$ $\qquad a = \frac{n_f^2}{n_c^2} \cdot \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2}$ TM

universal dispersion relation:

$$V\sqrt{1-b} = \nu\pi + \tan^{-1}\sqrt{b/(1-b)} + \tan^{-1}\sqrt{(b+a)/(1-b)}$$

(see fig. 3.14)

prev. example: $n_f = 1.5$, $n_s = 1.45$, $n_c = 1.40$, $h = 5\,\mu m$, $\lambda_0 = 1\,\mu m$

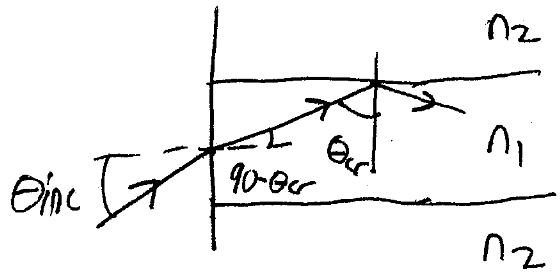
$V = 12.088$ $\quad a = 0.475$ $\quad$ (see fig. 3.15)

$b = 0.578, 0.813, 0.965$ $\quad$ cutoff when $b = 0$

or $V = \tan^{-1}\sqrt{a} + \nu\pi$

Numerical aperture - most applicable to MM

Max acceptance angle that stays above TIR in wg.



$$\sin\theta_{inc} = n_1 \sin(90-\theta_{cr})$$
$$= n_1 \cos(\theta_{cr})$$
$$= n_1\sqrt{1-\sin^2\theta_{cr}}$$
$$= n_1\sqrt{1-n_2^2/n_1^2}$$
$$= \sqrt{n_1^2-n_2^2}$$

$$NA = \sin\theta_{inc} = \sqrt{n_1^2-n_2^2}$$

$n_1 = 1.5 \quad n_2 = 1.4 \Rightarrow NA = 32^\circ$