1/14/09 ECE 6440 (1)

Maxwell's eqns

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{D} = \rho$$

E V/m

H A/m

D C/m^2 elec. flux density

B Wb/m^2 mag flux density

J A/m^2

ρ C/m^3

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{B} = \mu \vec{H}$$

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m

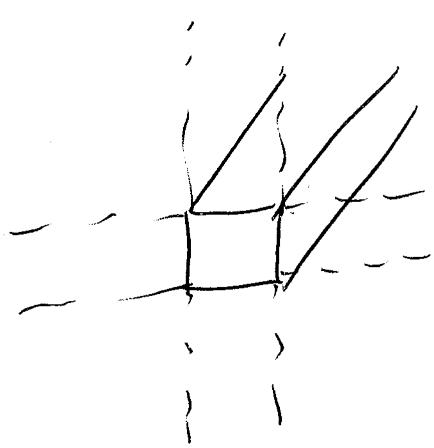
$\downarrow$

$$\vec{P} = \epsilon_0 \chi^{(1)} : E + \epsilon_0 \chi^{(2)} : E \cdot E + \ldots$$

$$\epsilon \equiv \epsilon_0 [1 + \chi^{(1)}] \qquad \chi = \chi_r + j\chi_i$$

$$\epsilon \to \epsilon_r + j\epsilon_i \qquad n \approx \sqrt{\epsilon_r/\epsilon_0}$$

The geometry we ultimately want to work with:



up to 9 different refractive indices

ϵ varies in space

solve as boundary value problem

wave equation

in dielectric media

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{D} = 0$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \qquad \nabla \cdot \vec{B} = 0$$

"reduce" to 2nd order eqn. in E:

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \left(\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \vec{H})$$

$$= -\mu_0 \frac{\partial^2 \vec{D}}{\partial t^2} = -\mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

use $\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$ to get

$$\nabla^2 \vec{E} - \nabla(\nabla \cdot \vec{E}) - \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

what's up with $\nabla(\nabla \cdot \vec{E})$?

use $\nabla \cdot \vec{D} = \nabla \cdot (\epsilon \vec{E}) = 0$

since ϵ vary's in space

$$\nabla \cdot (\epsilon \vec{E}) = \nabla \epsilon \cdot \vec{E} + \epsilon (\nabla \cdot \vec{E}) = 0$$

$$\Rightarrow \nabla \cdot \vec{E} = -\vec{E} \cdot \frac{\nabla \epsilon}{\epsilon} = -\vec{E} \cdot \nabla \ln \epsilon$$

if we assume ϵ const. in space, then $\nabla \cdot \vec{E} = 0$!

then wave eqn. is

$$\nabla^2 \vec{E} - \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Laplacian $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ cartesian

other forms for diff. coord. systems

$$\nabla^2 \vec{E} \Rightarrow \nabla^2 E_x \hat{x} + \nabla^2 E_y \hat{y} + \nabla^2 E_z \hat{z}$$

if no polarization mixing effects ($\nabla \cdot \vec{E} = 0$), then can solve for each component independently