Waveguide dispersion

consider TE mode, slab w.g.

$$\tan(hK) = \frac{\gamma_c + \gamma_s}{K\left(1 - \frac{\gamma_c \gamma_s}{K^2}\right)} \qquad K = \sqrt{k_0^2 n_f^2(\lambda) - \beta^2}$$

$$\gamma_{s,c} = \sqrt{\beta^2 - k_0^2 n_{s,c}^2(\lambda)}$$

can directly incorporate material dispersion, ~~but the~~ as we have to solve for β for every λ anyway

group delay

$$\tau_g = \frac{d\beta}{d\omega} = \frac{1}{c}\frac{d\beta}{dk} = \frac{1}{c} n_g \qquad n_g = \frac{d\beta}{dk} = \frac{c}{v_g}$$

$$v_g = \frac{d\omega}{d\beta}$$

dispersion

$$\Delta\tau = L D \Delta\lambda \qquad \Delta\tau = L\beta_2 \Delta\omega$$

$$D = -\frac{k^2}{2\pi c}\frac{d^2\beta}{dk^2} \qquad \beta_2 = \frac{d^2\beta}{d\omega^2}$$

$$= -\frac{\lambda_0}{c}\frac{d^2 n_{eff}}{d\lambda^2}$$

numerical differentiation

let's approximate $\beta'(\omega)$ and $\beta''(\omega)$

Taylor's series

a) $\beta(\omega + \delta\omega) \approx \beta(\omega) + \delta\omega\,\beta'(\omega) + \frac{(\delta\omega)^2}{2}\beta''(\omega)$

b) $\beta(\omega - \delta\omega) \approx \beta(\omega) - \delta\omega\,\beta'(\omega) + \frac{(\delta\omega)^2}{2}\beta''(\omega)$

1st derivative
subtract a - b

$$\beta(\omega+\delta\omega) - \beta(\omega-\delta\omega) \approx 2\,\delta\omega\,\beta'(\omega)$$

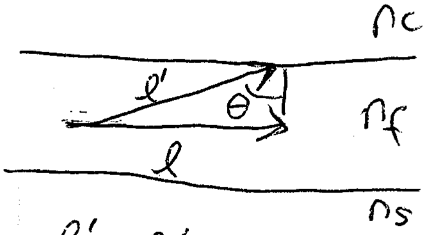
$$\Rightarrow \quad \beta'(\omega) \approx \frac{\beta(\omega+\delta\omega) - \beta(\omega-\delta\omega)}{2\,\delta\omega}$$

2nd derivative
add a + b

$$\beta(\omega+\delta\omega) + \beta(\omega-\delta\omega) \approx 2\beta(\omega) + (\delta\omega)^2\beta''(\omega)$$

$$\Rightarrow \quad \beta''(\omega) \approx \frac{\beta(\omega+\delta\omega) - 2\beta(\omega) + \beta(\omega-\delta\omega)}{(\delta\omega)^2}$$

Modal dispersion



$\longrightarrow$ = slow mode, largest β

$\nearrow$ = fast mode, smallest β

$l' = l/\cos\theta \quad \theta \le \theta_c$

$$\frac{\Delta\tau}{L} = \tau_{low} - \tau_{high} \qquad \tau_g = \frac{d\beta}{d\omega} \qquad \text{low: } \beta \approx k_0 n_f = \frac{\omega}{c} n_f$$

$$= \left(\frac{1}{c} n_f + \frac{\omega}{c} n_f'\right) - \left(\frac{1}{c} n_s + \frac{\omega}{c} n_s'\right) \qquad \text{high: } \beta \approx k_0 n_s = \frac{\omega}{c} n_s$$

$$= \frac{n_f - n_s}{c} + k_0 (n_f' - n_s') \approx \frac{n_f - n_c}{c}$$

$$\Delta\tau = \frac{n_f - n_c}{c} L$$ this works fine for large # modes

if only a few modes, need to solve for β's directly

also, this is extreme case, overestimate

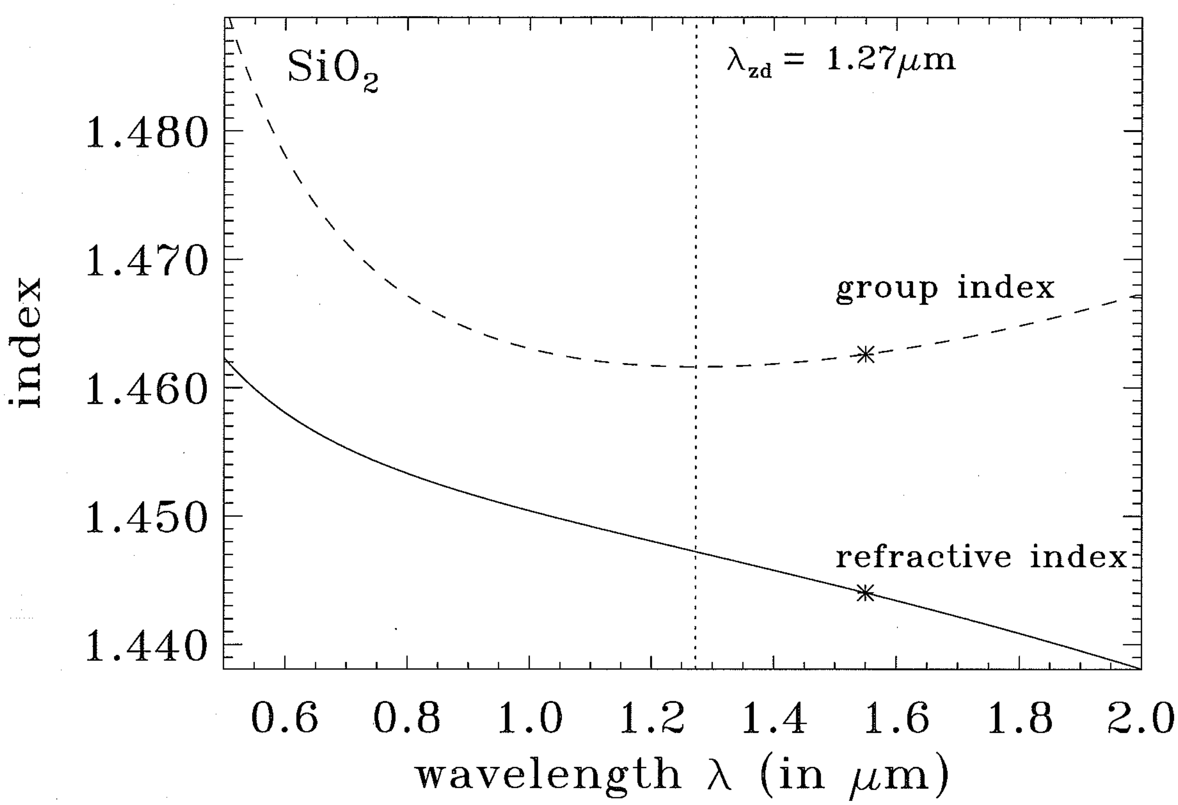
in reality, launched power couples non-uniformly into modes, so that total field

$$E_{tot} = \sum_{n=1}^{N} A_n E_n \qquad n = \text{mode \#}, \quad A_n = \text{amplitude}$$

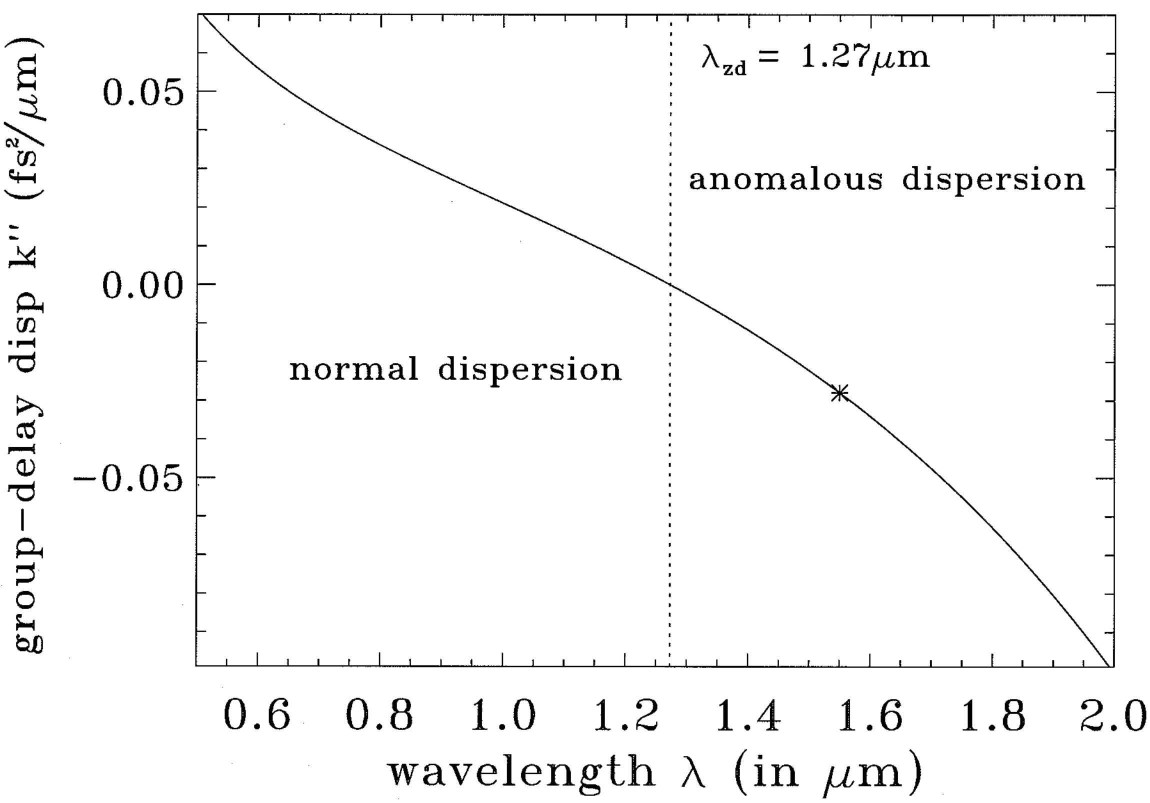
$$|E_{tot}|^2 \approx \sum_{n=1}^{N} |A_n|^2 |E_n|^2 \qquad I_{tot} \approx \sum_{n=1}^{N} |A_n|^2 I_n$$

$$\sum |A_n|^2 = 1$$

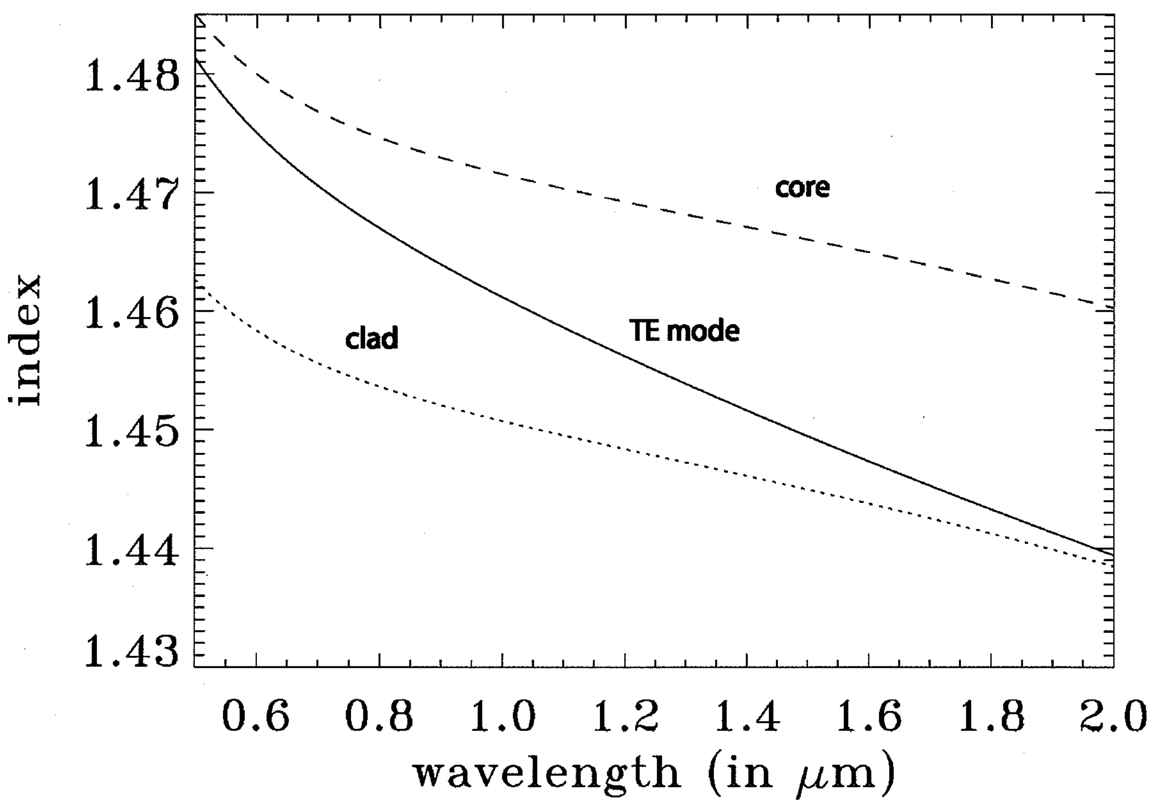
actual pulse spread $\Delta\tau$ based upon weighted average



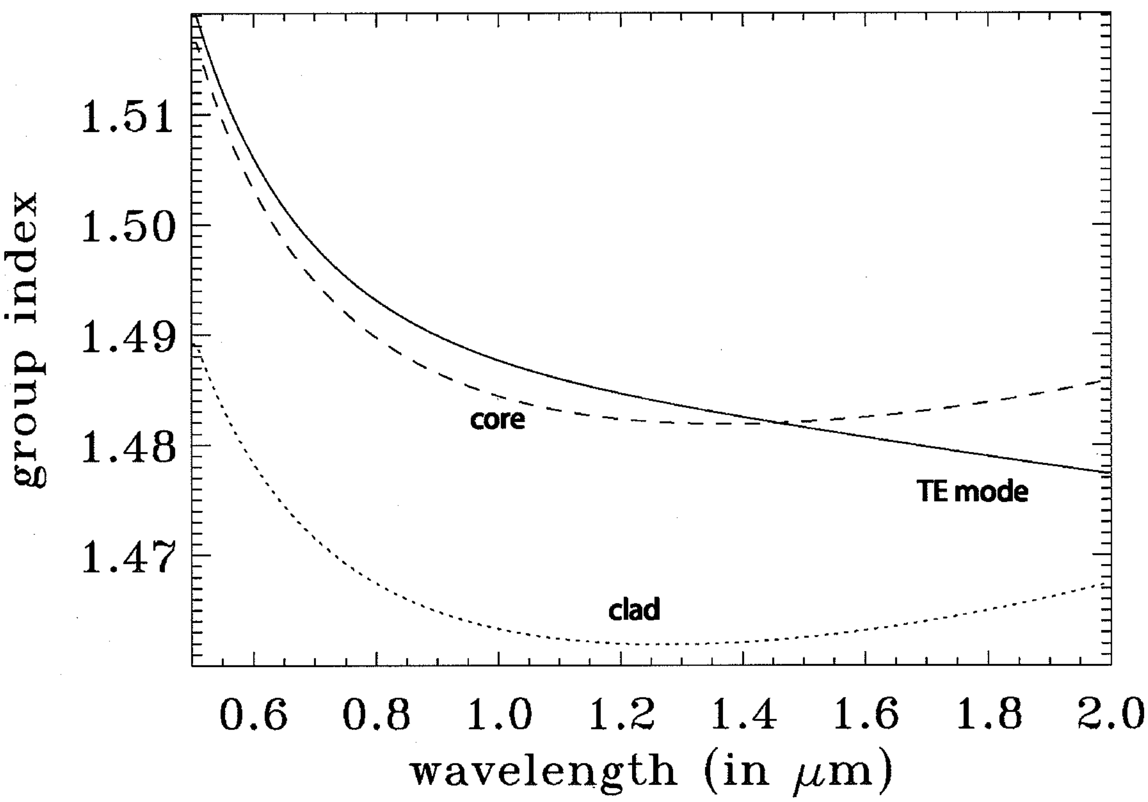
SiO2
λzd = 1.27μm
group index
refractive index
index
1.480
1.470
1.460
1.450
1.440
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
wavelength λ (in μm)

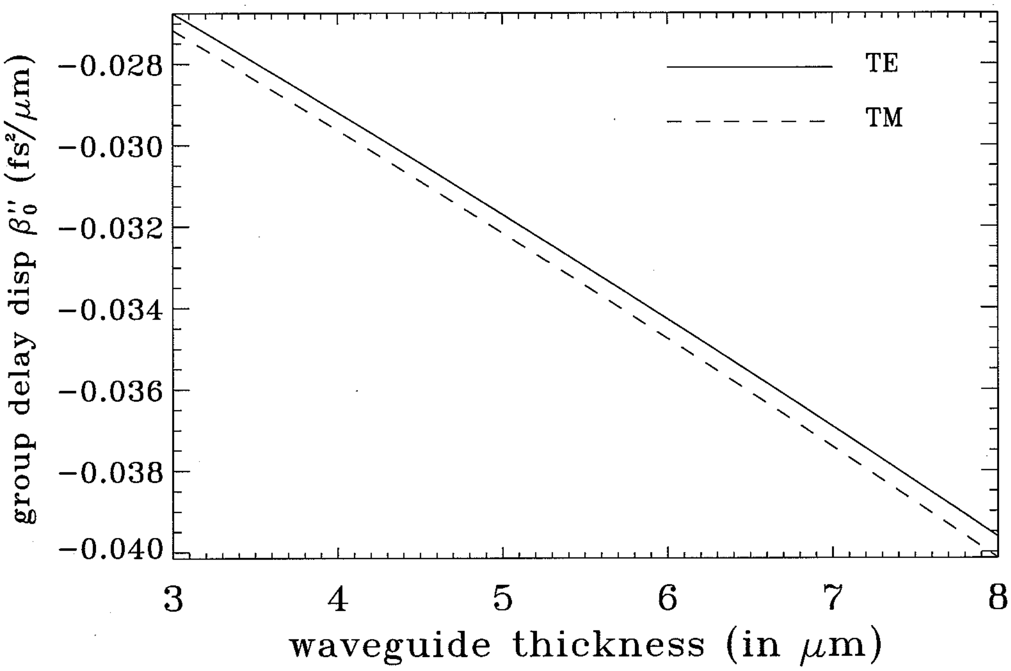


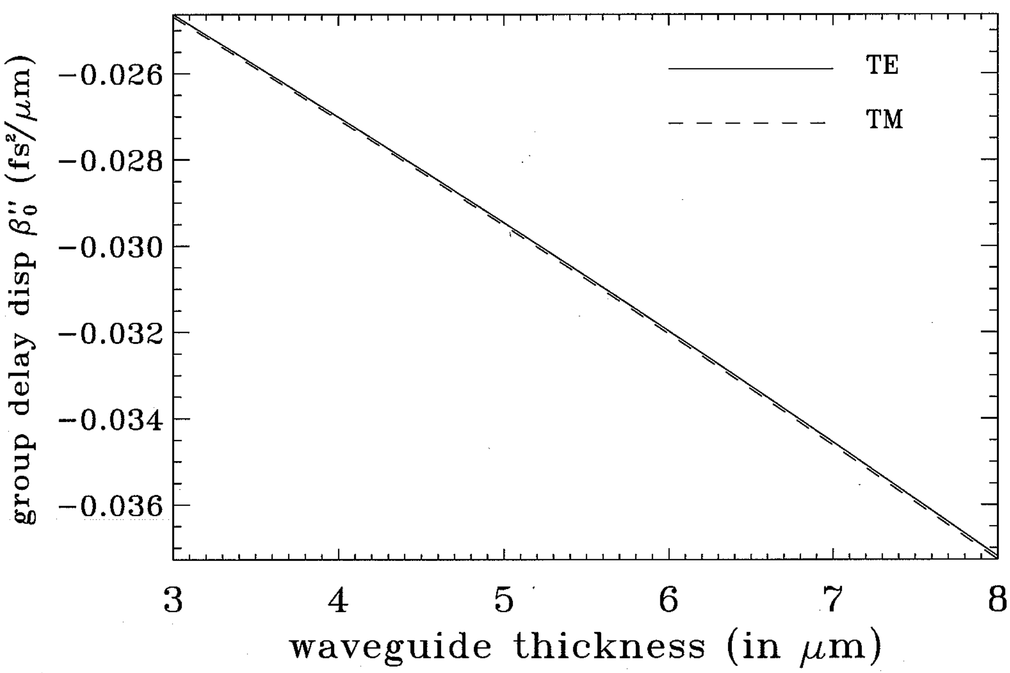
group-delay disp k'' (fs²/μm)
0.05
0.00
−0.05
λzd = 1.27μm
anomalous dispersion
normal dispersion
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
wavelength λ (in μm)



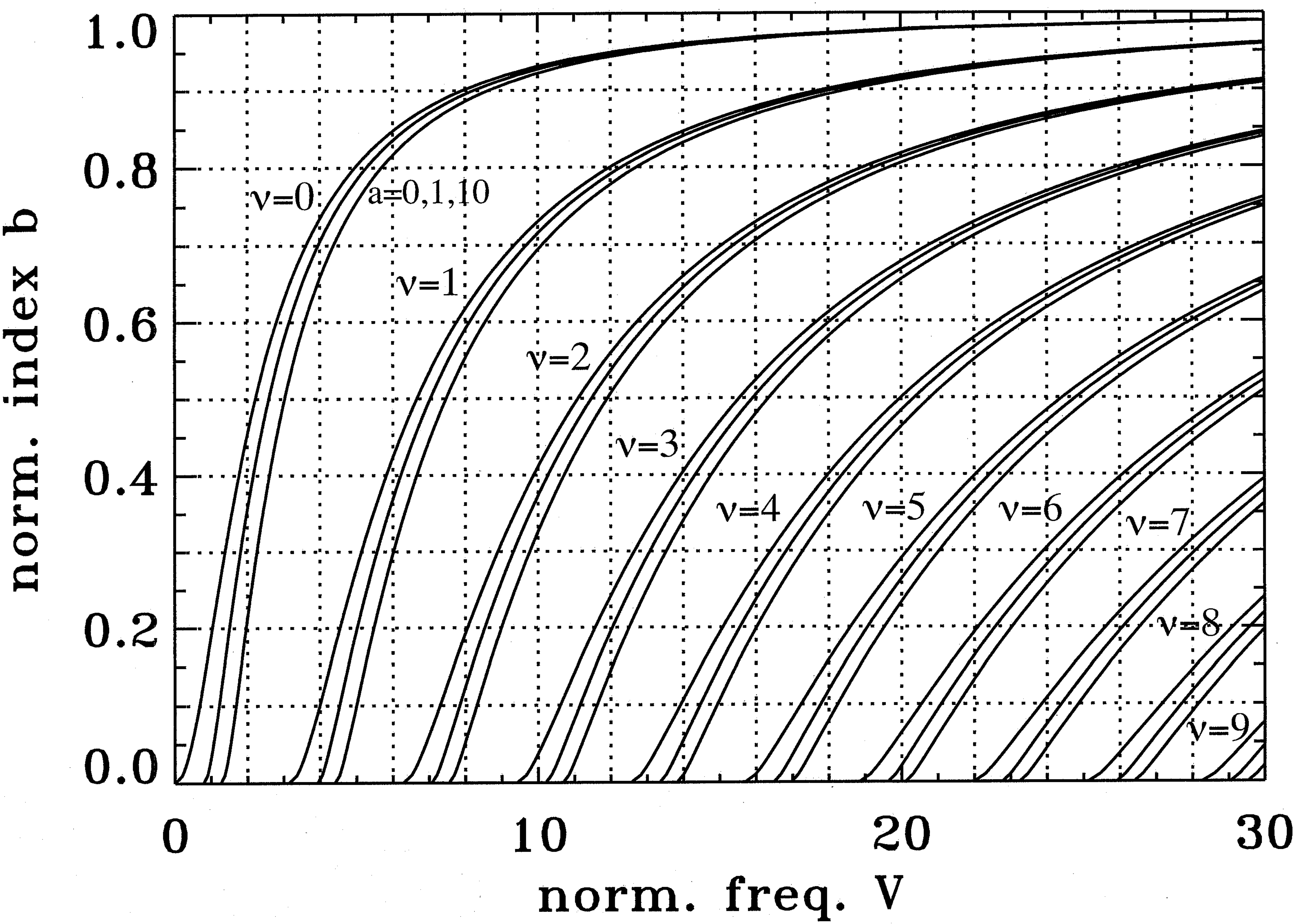
1.48
1.47
1.46
1.45
1.44
1.43
index
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
wavelength (in μm)
core
TE mode
clad


1.51
1.50
1.49
1.48
1.47
group index
core
TE mode
clad
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
wavelength (in μm)


TE
TM
−0.028
−0.030
−0.032
−0.034
−0.036
−0.038
−0.040
group delay disp β_0'' (fs²/μm)
3
4
5
6
7
8
waveguide thickness (in μm)



TE
TM
−0.026
−0.028
−0.030
−0.032
−0.034
−0.036
group delay disp β_0'' (fs²/μm)
3
4
5
6
7
8
waveguide thickness (in μm)


norm. index b
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
norm. freq. V
ν=0
a=0,1,10
ν=1
ν=2
ν=3
ν=4
ν=5
ν=6
ν=7
ν=8
ν=9