Direct Adaptive Cancellation of Periodic Disturbances for Multivariable Plants

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Abstract: A multivariable adaptive scheme is proposed that is designed to reject periodic disturbances of unknown frequency at multiple sensor locations. The case of square systems is considered, as well as the common case where there are more sensors than actuators (overdetermined systems). The disturbances are assumed to contain multiple harmonics associated with a single fundamental frequency, as is the case if the source of the disturbances is a rotating machine. Accordingly, the control inputs are selected to be periodic signals with a frequency determined by a single frequency estimate. Although the adaptive scheme is highly nonlinear, approximations yield an analysis that enables a linear time-invariant design of the algorithm. The analysis also gives useful information about the dynamic behavior of the system and the selection of the design parameters. Experimental results show that the proposed scheme is able to significantly reduce acoustic noises with constant or time-varying frequency at multiple locations in an enclosure.

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1. Introduction

An active disturbance control system rejects or reduces disturbances by introducing signals that are opposite in sign to the disturbance signals. There are two basic types of active control systems: feedforward and feedback control systems. The effectiveness of feedforward control methods depends on the availability of reference signals that are highly correlated with the disturbance signals. It is suitable for applications where a reference signal may be obtained from a non-acoustic sensor such as a tachometer or an accelerometer, or from a detection sensor placed upstream of the control sources [1]. However, it is not always feasible to obtain a reference signal that is well correlated with the disturbance signals, giving time-advanced information. In such cases, feedback control is needed, where the signals driving the control actuators are obtained directly from the error sensors, rather than from an independent reference sensor.

The general block diagram of a multivariable feedback disturbance control system is shown in Fig. 1. In the figure, $\mathbf{n}(t)$ is the vector composed of the disturbance signals at multiple sensor locations, $\mathbf{u}(t)$ is the vector of the control signals applied to actuators, and $\mathbf{e}(t)$ is the vector of the error signals measured by the sensors. The physical dynamics of the control signals propagating from the actuators to the sensors together with the response of data converters and anti-aliasing and reconstruction filters constitutes the plant $\mathbf{P}(s)$. The controller is often made adaptive to track the slowly time-varying properties of the disturbances and to compensate for slow changes in the plant response.

Figure 1: A multivariable feedback disturbance control system.

The emphasis of this paper is on multivariable feedback control of periodic disturbances which contain multiple harmonics of a single fundamental frequency. Examples of periodic disturbances are acoustic noise and vibration originating from the rotation of an engine, compressor, fan, or propeller. The frequency of the disturbance is not known *a priori* and may be slowly time-varying due to slow changes in rotational speed. The amplitudes of the constituent sinusoids of the disturbances may also be varying slowly.

A multivariable control system is needed to effectively control sound over extended regions of space. Another reason for the development of a multivariable disturbance control system is to deal with possible singularities of a single control channel. If the frequency response of a single channel plant $P(j\omega)$ at some frequency is zero, the system is ineffective at controlling the signal component at that frequency, regardless of the control structure or algorithm. This problem may be overcome by using multiple control sources and channels that do not have common zeros in their transfer functions.

There is a relatively large volume of literature on active noise control with a reference signal (*feedforward control*) and on the feedback control of periodic sources of *known frequency*. Few algorithms exist for the rejection of periodic disturbances of unknown frequency using feedback, including applications to multi-channel active noise control systems. An early reference is the patent of Chaplin & Smith [2], which describes in broad terms the concept of an active noise control system where the frequency of the disturbance is estimated using a phase-locked loop. An analysis of one of the possible implementations was presented in [3]. In adaptive control, the terminology "indirect" usually applies to an algorithm where the parameters of the system are estimated, and inserted in a control algorithm using a design procedure that assumes that the estimates are exact. In analogy, approaches where the frequency of the disturbance is estimated, and then applied in a scheme for the cancellation of disturbances of known frequency are called indirect in this paper, as in [4]. Recently, there have been a few attempts at implementing indirect algorithms in active noise control. References [5], [6], [7] and [8] report on experimental results obtained with adaptive notch filters for frequency estimation.

The approach used in this paper is significantly different from the indirect algorithms and originates from [4]. The scheme is called direct because a nonlinear update law is designed to directly adjust all the parameters of the disturbance cancellation signal so that they converge towards the desired values. The scheme may be viewed as an extension of a phase-locked loop, where the plant (the system originating from the propagation of sound from the speaker to the microphone) is placed *inside* the phase-locked loop. One of the advantages of this approach is that, although the overall system is highly nonlinear, an approximate analysis around the nominal parameters yields a linear time-invariant system, so that the parameters of the system can be adjusted to produce predictable transient behavior.

The direct approach of [4] was modified to control periodic noise with multiple harmonics in [9] and experimental results were presented. In this paper, the algorithm is extended to a multivariable disturbance control system. A modified feedback design is also proposed for the adaptation of the frequency and phases of the control signals, resulting in improved transient behavior. The control algorithm is applied to the reduction of periodic acoustic noise in a small room, and experimental results demonstrate excellent practical performance of the system for constant and time-varying noise frequency.

2. Multivariable Disturbance Control

2.1 Problem Statement

Assume that the effect of the disturbances is additive and that the channels from the actuators to the sensors can be described by stable, linear time-invariant systems with transfer functions $P_{li}(s)$, where $l = 1, ..., L$ stands for control input #1, ..., L, and $i = 1$, ..., I stands for residual error #1, 2, ..., I. It is assumed that the number of error sensors is greater than or equal to the number of control inputs in the system, i.e., $I \geq L$. This is a typical situation in active noise control systems and many other applications where sensors are cheaper than actuators. Adjustments may be made to solve the case $I < L$, but are not considered in this paper.

The system transfer function matrix $\mathbf{P}(s)$ has elements $P_{li}(s)$, *where* l *specifies the column and* *i specifies the row.* It is assumed that suitable locations for the control sources and the sensors are chosen, so that the system frequency response matrix is of full rank (i.e., $\mathbf{P}(j\omega)$ has L linearly independent rows) at the frequencies of the disturbance signals. Fig. 2 shows the structure of the multi-channel disturbance control system, where L control actuators are used to control disturbance signals at I sensor locations. The symbols $u_l(t)$, $e_i(t)$, and $n_i(t)$ are, respectively, the control signals, the sensor signals (or the plant outputs), and the disturbance signals at the sensor locations. The symbol $p_{li}(t)$ for $l = 1, ..., L$ and $i = 1$, ..., I is the impulse response of the channel transfer function $P_{li}(s)$. Defining the vector of error signals $\mathbf{e}(t)$, we have

$$\mathbf{e}(t) = \begin{bmatrix} e_1(t) \\ \vdots \\ e_I(t) \end{bmatrix} = \begin{bmatrix} p_{11}(t) & \dots & p_{L1}(t) \\ \vdots & \vdots & \vdots \\ p_{1I}(t) & \dots & p_{LI}(t) \end{bmatrix} * \begin{bmatrix} u_1(t) \\ \vdots \\ u_L(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ \vdots \\ n_I(t) \end{bmatrix}, \tag{1}$$

where $*$ denotes linear convolution. The disturbances $n_i(t)$ with $i = 1$, ..., I for different sensor locations are assumed to contain multiple harmonics associated with a single fundamental frequency, as is the case if the source of the disturbances is a rotating machine. The disturbance signals are represented by

$$n_i(t) = \sum_{k=1}^{K} \left\{ \pi_{i,k}^{\mathrm{c}} \cos\left[k\alpha_{\mathrm{d}}(t)\right] - \pi_{i,k}^{\mathrm{s}} \sin\left[k\alpha_{\mathrm{d}}(t)\right] \right\}, \; i = 1, 2, ..., I \tag{2}$$

where $\alpha_{\mathrm{d}}(t) = \omega_{\mathrm{d}} \cdot t$, and ω_{d} is the fundamental frequency of the disturbances. The initial phases and the amplitudes of the disturbance components are parameterized as the amplitude parameters $\pi_{i,k}^{\mathrm{c}}$ and $\pi_{i,k}^{\mathrm{s}}$. For the purpose of analysis, ω_{d}, $\pi_{i,k}^{\mathrm{c}}$, and $\pi_{i,k}^{\mathrm{s}}$ are assumed constant. In practice, the parameters may be slowly-varying. The order of the highest harmonic to be cancelled, K, is assumed to be finite and known.

The objective of the control system is to generate control signals $u_l(t)$ such that the effects of the disturbance signals at the sensor locations are cancelled through destructive interference. If there are more sensors than actuators (i.e., $I > L$, and the system is said to be overdetermined), and exact cancellation may not be possible even under ideal conditions.

2.2 Optimal Solution and Problem Reformulation

Figure 2: Multivariable disturbance control problem.

The optimal solution depends on the cost function to be minimized. In this paper, the time-averaged sum of the squared errors is considered as a cost function: $J(t) = \mathrm{AVG}\left[\mathbf{e}^T(t)\mathbf{e}(t)\right]$. Given that the plant $\mathbf{P}(s)$ is assumed to be linear, the control inputs are selected to contain every frequency component of the disturbances, with

$$u_l(t) = \sum_{k=1}^{K} \left\{\theta^{\mathrm{c}}_{l,k} \cos\left[k\alpha(t)\right] - \theta^{\mathrm{s}}_{l,k} \sin\left[k\alpha(t)\right]\right\}, \; l = 1, 2, ..., L, \tag{3}$$

where $\alpha(t) = \omega t$, and the fundamental frequency of the control signals $\omega = \omega_{\mathrm{d}}$ for optimal operation. The optimal amplitude parameters $\theta^{\mathrm{c}}_{l,k}$ and $\theta^{\mathrm{s}}_{l,k}$ depend on the plant response at the disturbance frequencies and on the amplitude parameters $\pi^{\mathrm{c}}_{i,k}$ and $\pi^{\mathrm{s}}_{i,k}$ of the disturbances.

To present the analysis in a compact form, we group the cos and sin amplitude parameters of the control signals in a vector for each harmonic

$$\boldsymbol{\theta}_k = \begin{bmatrix} \theta^{\mathrm{c}}_{1,k} & ... & \theta^{\mathrm{c}}_{L,k} & \theta^{\mathrm{s}}_{1,k} & ... & \theta^{\mathrm{s}}_{L,k} \end{bmatrix}^T, \; k = 1, 2, ..., K. \tag{4}$$

Similarly, the amplitude parameter vector of the disturbance signals is defined as

$$\boldsymbol{\pi}_k = \begin{bmatrix} \pi^{\mathrm{c}}_{1,k} & ... & \pi^{\mathrm{c}}_{I,k} & \pi^{\mathrm{s}}_{1,k} & ... & \pi^{\mathrm{s}}_{I,k} \end{bmatrix}^T, \; k = 1, 2, ..., K. \tag{5}$$

For constant amplitude parameters of the control signals, the response of the plant to the signals $u_l(t)$ may be replaced by its steady-state counterpart, since transient components do not affect the cost function. In this case, the sensor signals are given by

$$\mathbf{e}(t) = \sum_{k=1}^{K} \left[\mathbf{Q}_k(t)(2\mathbf{G}_k\boldsymbol{\theta}_k + \boldsymbol{\pi}_k)\right], \tag{6}$$

where the $I \times 2I$ matrix $\mathbf{Q}_k(t)$ is defined to be

$$\mathbf{Q}_k(t) = \begin{bmatrix} \cos[k\alpha(t)]\mathbf{I}_{I\times I} & -\sin[k\alpha(t)]\mathbf{I}_{I\times I} \end{bmatrix}, \tag{7}$$

and $\mathbf{I}_{I\times I}$ is an $I \times I$ identity matrix. The $2I \times 2L$ matrices $\mathbf{G}_k$ are given by

$$\mathbf{G}_k = \frac{1}{2}\begin{bmatrix} \mathbf{P}_k^{\mathrm{Re}} & -\mathbf{P}_k^{\mathrm{Im}} \\ \mathbf{P}_k^{\mathrm{Im}} & \mathbf{P}_k^{\mathrm{Re}} \end{bmatrix}, \tag{8}$$

where $\mathbf{P}_k^{\mathrm{Re}}$ and $\mathbf{P}_k^{\mathrm{Im}}$ are the real and imaginary parts of the plant response at the frequency of the kth harmonic of the control signals, i.e.,

$$\mathbf{P}(jk\omega) = \mathbf{P}_k^{\mathrm{Re}} + j\mathbf{P}_k^{\mathrm{Im}} = \begin{bmatrix} P_{11}(jk\omega) & \cdots & P_{L1}(jk\omega) \\ \vdots & \vdots & \vdots \\ P_{1I}(jk\omega) & \cdots & P_{LI}(jk\omega) \end{bmatrix}. \tag{9}$$

Using the above definitions, one may compute the cost function given by

$$J = \sum_{k=1}^{K} \left(\frac{1}{2}\boldsymbol{\pi}_k^T\boldsymbol{\pi}_k + \boldsymbol{\pi}_k^T\mathbf{G}_k\boldsymbol{\theta}_k + \boldsymbol{\theta}_k^T\mathbf{G}_k^T\boldsymbol{\pi}_k + 2\boldsymbol{\theta}_k^T\mathbf{G}_k^T\mathbf{G}_k\boldsymbol{\theta}_k \right). \tag{10}$$

The derivatives of J with respect to $\boldsymbol{\theta}_k$ are set to zero, which leads to the optimal amplitude parameters of the control signals

$$\boldsymbol{\theta}_k^0 = -\frac{1}{2}(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T\boldsymbol{\pi}_k, \tag{11}$$

with

$$\boldsymbol{\theta}_k^0 = \begin{bmatrix} \theta_{1,k}^{\mathrm{c}0} & ... & \theta_{L,k}^{\mathrm{c}0} & \theta_{1,k}^{\mathrm{s}0} & ... & \theta_{L,k}^{\mathrm{s}0} \end{bmatrix}^T, \tag{12}$$

for $k = 1, 2, ..., K$, and the optimal control inputs are defined accordingly.

The above result shows that the control of each frequency component of the disturbances is independent of the others in steady-state. With the optimal control inputs, the error signals in steady-state are

$$\mathbf{e}(t)=\sum_{k=1}^{K}\left\{\mathbf{Q}_k(t)\left[-\mathbf{G}_k(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T+\mathbf{I}_{2I\times 2I}\right]\boldsymbol{\pi}_k\right\}, \tag{13}$$

where $\mathbf{I}_{2I\times 2I}$ is a $2I\times 2I$ identity matrix. For overdetermined systems, exact cancellation of the disturbances is generally not possible, even under ideal conditions. However, for fully-determined systems ($I=L$), the residual error signals are zero in steady-state, since the matrix $\mathbf{G}_k$ is non-singular due to the full-rank assumption of $\mathbf{P}(j\omega)$. In this case, the disturbances are completely rejected (similar results were also reported in [10]).

We model the disturbance signals $\mathbf{n}(t)$ as the sum of two sets of disturbances, one that is reflected at the input of the plant and can be rejected completely, the other containing the disturbances at the plant output (sensor locations). The reflected disturbance sources are

$$d_l(t)=\sum_{k=1}^{K}\left\{\theta_{l,k}^{\mathrm{c}0}\cos\left[k\alpha_{\mathrm{d}}(t)\right]-\theta_{l,k}^{\mathrm{s}0}\sin\left[k\alpha_{\mathrm{d}}(t)\right]\right\},\ \text{for } l=1,2,...,L. \tag{14}$$

The disturbances modeled at the output of the plant are equivalent to the residual disturbances in steady-state with the optimal control input. The vector of the output disturbances is given by

$$\mathbf{r}(t)=\sum_{k=1}^{K}\left\{\mathbf{Q}_k(t)\left[-\mathbf{G}_k(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T+\mathbf{I}_{2I\times 2I}\right]\boldsymbol{\pi}_k\right\}.$$

Fig. 3 shows the reformulated control problem. By reformulating the disturbance signals in this way, we will be able to show in Section 3 that the output disturbances $\mathbf{r}(t)$ for overdetermined systems do not affect the convergence of the adaptive parameters of the proposed control scheme, and that the input disturbances $\mathbf{d}(t)$ are canceled using the proposed feedback design.

3. Direct Adaptive Scheme

3.1 Overall Scheme

We consider a control scheme that iteratively adjusts the control signals in order to minimize the cost function. Adaptation is needed, as the amplitudes and frequency of the disturbances are unknown and may change slowly with time. The disturbance sources reflected at the plant inputs are assumed to be

$$d_l(t)=\sum_{k=1}^{K}m_{\mathrm{d},l,k}\cos\left[\alpha_{\mathrm{d},l,k}(t)\right],\ l=1,2,...,L, \tag{15}$$

Figure 3: Reformulated multivariable disturbance control problem.

where $\dot{\alpha}_{\mathrm{d},l,k}(t) = k\omega_{\mathrm{d}}$. Note that for the purpose of the adaptive scheme to be discussed, it is useful to reformulate the disturbances in terms of their amplitude/phase parameters rather than the cos/sin components used earlier. The control signals contain every frequency component of the disturbance signals and are given by

$$u_l(t) = \sum_{k=1}^{K} m_{l,k}(t) \cos\left[\alpha_{l,k}(t)\right], \; l = 1, 2, ..., L, \tag{16}$$

where the amplitudes and angles have nominal values $m_{l,k}^0 = -m_{\mathrm{d},l,k}$ and $\alpha_{l,k}^0 = \alpha_{\mathrm{d},l,k}$.

Fig. 4 shows the structure of the direct scheme for multivariable control of periodic disturbances. The error signal vector $\mathbf{e}(t)$ is first multiplied by $\cos\left[\alpha_{l,k}(t)\right]$ and $-\sin\left[\alpha_{l,k}(t)\right]$, where $\alpha_{l,k}(t)$ is the angle of the kth harmonic of the lth control input, for $l = 1, 2, ..., L$, and $k = 1, 2, ..., K$. In the figure, plant compensation removes the effect of the plant frequency response on the measured signals and consists of the following matrix multiplication

$$\begin{bmatrix} y_{\mathrm{c},l,k}(t) \\ y_{\mathrm{s},l,k}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{k,l} \\ \mathbf{H}_{k,(L+l)} \end{bmatrix} \begin{bmatrix} \mathbf{e}(t)\cos\left[\alpha_{l,k}(t)\right] \\ -\mathbf{e}(t)\sin\left[\alpha_{l,k}(t)\right] \end{bmatrix}, \text{ with } l = 1, 2, ..., L, \text{ and } k = 1, 2, ..., K. \tag{17}$$

The row vectors $\mathbf{H}_{k,l}$ and $\mathbf{H}_k(L+l)$ are the lth and $(L+l)$th rows of the $(2L \times 2I)$ matrix $\mathbf{H}_k$

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{H}_{k,1} \\ \mathbf{H}_{k,2} \\ \vdots \\ \mathbf{H}_{k,2L} \end{bmatrix} = (\hat{\mathbf{G}}_k^T \hat{\mathbf{G}}_k)^{-1} \hat{\mathbf{G}}_k^T. \tag{18}$$

The $(2I \times 2L)$ matrix $\hat{\mathbf{G}}_k$ is the estimate of the plant matrix $\mathbf{G}_k$, which is defined in Section 2.2. In practice, $\mathbf{G}_k$ is estimated during a preliminary training phase using appropriate input signals.

Figure 4: Direct scheme for multivariable disturbance cancellation.

The variables $y_{\mathrm{c},l,k}(t)$ and $y_{\mathrm{s},l,k}(t)$ ($l = 1, 2, ..., L$, and $k = 1, 2, ..., K$) are used in the feedback design to calculate the amplitudes, the fundamental frequency ω, and the relative phases $\phi_{l,k}$ of the control signals, where $l = 2, ...L$ for $k = 1$, and $l = 1, 2, ...L$ for $k = 2, ..., K$. The feedback design is given in Section 3.3.

3.2 Approximate Analysis

The analysis of the system is similar to that in [4] and [9], and inspired by analysis techniques used for phase-locked loops. We first assume that the output disturbances r_i, with $i = 1, 2, ..., I$ are zero. The responses of the plant to the control signals and the disturbance sources are approximated by their steady-state outputs, and the high-frequency components resulting from the multiplication are discarded. Under these assumptions, we have

$$\begin{bmatrix} \mathbf{e}(t)\cos[\alpha_{l,k}(t)] \\ -\mathbf{e}(t)\sin[\alpha_{l,k}(t)] \end{bmatrix} = \mathbf{G}_k \begin{bmatrix} \mathbf{m}_k(t)\odot\cos[\boldsymbol{\alpha}_k(t)-\alpha_{l,k}(t)] - \mathbf{m}_k^0\odot\cos[\boldsymbol{\alpha}_k^0(t)-\alpha_{l,k}(t)] \\ \mathbf{m}_k(t)\odot\sin[\boldsymbol{\alpha}_k(t)-\alpha_{l,k}(t)] - \mathbf{m}_k^0\odot\cos[\boldsymbol{\alpha}_k^0(t)-\alpha_{l,k}(t)] \end{bmatrix}, \quad (19)$$

with

$$\begin{aligned} \mathbf{m}_k(t) &= [m_{1,k}(t), ..., m_{L,k}(t)]^T, \ \boldsymbol{\alpha}_k(t) = [\alpha_{1,k}(t), ..., \alpha_{L,k}(t)]^T, \\ \mathbf{m}_k^0 &= [m_{1,k}^0, ..., m_{L,k}^0]^T, \ \boldsymbol{\alpha}_k^0(t) = [\alpha_{1,k}^0(t), ..., \alpha_{L,k}^0(t)]^T, \end{aligned} \quad (20)$$

and the operator $\odot$ is defined to be such that

$$\mathbf{m}_k(t)\odot\cos[\boldsymbol{\alpha}_k(t)-\alpha_{l,k}(t)] = \begin{bmatrix} m_{1,k}(t)\cos[\alpha_{1,k}(t)-\alpha_{l,k}(t)] \\ \vdots \\ m_{L,k}(t)\cos[\alpha_{L,k}(t)-\alpha_{l,k}(t)] \end{bmatrix}. \quad (21)$$

Then

$$\mathbf{H}_k \begin{bmatrix} \mathbf{e}(t)\cos[\alpha_{l,k}(t)] \\ -\mathbf{e}(t)\sin[\alpha_{l,k}(t)] \end{bmatrix} = \mathbf{H}_k\mathbf{G}_k \begin{bmatrix} \mathbf{m}_k(t)\odot\cos[\boldsymbol{\alpha}_k(t)-\alpha_{l,k}(t)] - \mathbf{m}_k^0\odot\cos[\boldsymbol{\alpha}_k^0(t)-\alpha_{l,k}(t)] \\ \mathbf{m}_k(t)\odot\sin[\boldsymbol{\alpha}_k(t)-\alpha_{l,k}(t)] - \mathbf{m}_k^0\odot\cos[\boldsymbol{\alpha}_k^0(t)-\alpha_{l,k}(t)] \end{bmatrix}. \quad (22)$$

If there is no modeling error and the frequency estimate is equal to the true value, $\hat{\mathbf{G}}_k = \mathbf{G}_k$. Consequently, $\mathbf{H}_k\mathbf{G}_k$ becomes the identity matrix, and

$$\mathbf{H}_k \begin{bmatrix} \mathbf{e}(t)\cos[\alpha_{l,k}(t)] \\ -\mathbf{e}(t)\sin[\alpha_{l,k}(t)] \end{bmatrix} = \begin{bmatrix} \mathbf{m}_k(t)\odot\cos[\boldsymbol{\alpha}_k(t)-\alpha_{l,k}(t)] - \mathbf{m}_k^0\odot\cos[\boldsymbol{\alpha}_k^0(t)-\alpha_{l,k}(t)] \\ \mathbf{m}_k(t)\odot\sin[\boldsymbol{\alpha}_k(t)-\alpha_{l,k}(t)] - \mathbf{m}_k^0\odot\cos[\boldsymbol{\alpha}_k^0(t)-\alpha_{l,k}(t)] \end{bmatrix}. \quad (23)$$

The variables $y_{\mathrm{c},l,k}(t)$ and $y_{\mathrm{s},l,k}(t)$ are given by

$$\begin{bmatrix} y_{\mathrm{c},l,k}(t) \\ y_{\mathrm{s},l,k}(t) \end{bmatrix} = \begin{bmatrix} m_{l,k}(t) - m_{l,k}^0\cos\left[\alpha_{l,k}^0(t)-\alpha_{l,k}(t)\right] \\ m_{\mathrm{d},l,k}\sin\left[\alpha_{l,k}^0(t)-\alpha_{l,k}(t)\right] \end{bmatrix} \simeq \begin{bmatrix} m_{l,k}(t) - m_{l,k}^0 \\ -m_{\mathrm{d},l,k}\left[\alpha_{l,k}(t)-\alpha_{l,k}^0(t)\right] \end{bmatrix} \quad (24)$$

for $l = 1, 2, ..., L$, and $k = 1, 2, ..., K$. Equation (24) shows that the variables $y_{\mathrm{c},l,k}(t)$ and $y_{\mathrm{s},l,k}(t)$ contain information about the amplitude and angle parameters of the disturbance sources. The information is free of the multivariable plant response effects. The paths from the parameters $m_{l,k}(t)$ and $\alpha_{l,k}(t)$ to the variables $y_{\mathrm{c},l,k}(t)$ and $y_{\mathrm{s},l,k}(t)$ are decoupled from one another and are independent for each harmonic. The relationship is approximately linear when $\alpha_{l,k}(t)$ is close to the nominal value $\alpha^0_{l,k}(t)$, as is shown by the second equation of (24).

3.3 Feedback Design

The adaptive feedback can now be designed according to the linear relationship between the parameters $m_{l,k}(t)$ and $\alpha_{l,k}(t)$ to the variables $y_{\mathrm{c},l,k}(t)$ and $y_{\mathrm{s},l,k}(t)$. The amplitudes of the control signals are updated according to

$$\dot{m}_{l,k}(t) = -g_{\mathrm{m},l,k} \cdot y_{\mathrm{c},l,k}(t), \text{ with } l = 1, 2, ..., L,\ k = 1, 2, ..., K \tag{25}$$

where $g_{\mathrm{m},l,k} > 0$ is an arbitrary adaptation gain. As a result, the dynamics of the amplitude loops are given by

$$\dot{m}_{l,k}(t) = -g_{\mathrm{m},l,k} \left[m_{l,k}(t) - m^0_{l,k} \right], \tag{26}$$

for $l = 1, 2, ..., L$ and $k = 1, 2, ..., K$, and the parameter estimates $m_{l,k}(t)$ converge to their ideal values as first-order systems with poles at $s = -g_{\mathrm{m},l,k}$.

Since the disturbance signals have the same fundamental frequency ω_{d}, it would be helpful to have a single frequency estimate $\omega(t)$ for all the signals. In this scheme, the fundamental frequency $\omega(t)$ is estimated using the variable $y_{\mathrm{s},1,1}(t)$, which corresponds to the fundamental component of the plant input #1. The feedback design with an adaptation gain g_ω is given by

$$\dot{\omega}(t) = g_\omega \cdot y_{\mathrm{s},1,1}(t), \tag{27}$$

since

$$y_{\mathrm{s},1,1}(t) = -m_{\mathrm{d},1,1} \left[\alpha_{1,1}(t) - \alpha^0_{1,1}(t) \right].$$

The phase of the fundamental component of the control input #1 $\alpha_{1,1}(t)$ is constructed via

$$\dot{\alpha}(t) \quad = \quad \omega,\ \alpha(0) = 0,$$

$$\alpha_{1,1}(t) \quad = \quad \alpha(t) + K_\mathrm{f}\omega, \tag{28}$$

where the design parameter K_f is positive.

Fig. 5 shows the linear approximation of the fundamental frequency update loop. Note that the phase $\alpha_{1,1}(t)$ is not purely integrated from the frequency $\omega(t)$, as was done in the previous designs of phase adaptation [4] [9]. The proportional gain K_f improves the dynamic response of the algorithm and simplifies its design. The dynamics of the linear loop are those of a second-order system with poles determined by the roots of

$$s^2 + g_\omega m_{\mathrm{d},1,1} K_\mathrm{f} s + g_\omega m_{\mathrm{d},1,1} = 0. \tag{29}$$

Stability is guaranteed as the values of K_f and $g_\omega m_{\mathrm{d},1,1}$ are both positive. The estimate $\omega(t)$ converges to its nominal value $\omega^0 = \omega_\mathrm{d}$. The relative phase of the fundamental of the first control signal is determined indirectly through the time history of $\omega(t)$. Specifically, the phase $\alpha_{1,1}(t)$ is equal to $\alpha^0_{1,1}(t) = \alpha_{\mathrm{d},1,1}(t)$ if $\omega(t) = \omega_\mathrm{d}$ at some time t (possibly infinite), and

$$\int_0^t [\omega(\tau) - \omega_\mathrm{d}]\, d\tau + K_\mathrm{f}\omega_\mathrm{d} = \alpha_{\mathrm{d},1,1}(0). \tag{30}$$

Figure 5: Approximate frequency update loop.

The phases of other components of the control inputs are obtained from the fundamental frequency estimate, taking into account the integer multiplicative relation between the fundamental frequency and the harmonic frequencies. Specifically, we let

$$\alpha_{l,k}(t) = k\alpha(t) + \phi_{l,k}(t), \tag{31}$$

where $l = 2, ...L$ for $k = 1$, and $l = 1, 2, ...L$ for $k = 2, ..., K$. For these phases, the parameters to be adapted are the relative phases $\phi_{l,k}(t)$. The nominal values for $\phi_{l,k}(t)$ are

$$\phi^0_{l,k} = k \int_0^t \left[\omega(\tau) - \omega_{\mathrm{d}}\right] d\tau + \alpha_{\mathrm{d},l.k}(0). \tag{32}$$

After the frequency loop converges, the linearized dynamics from the relative phases $\phi_{l,k}(t)$ to the variables $y_{\mathrm{s},l,k}(t)$ are given by

$$y_{\mathrm{s},l,k}(t) = -m_{\mathrm{d},l,k} \cdot \left[\phi_{l,k}(t) - \phi^0_{l,k}\right]. \tag{33}$$

with $l = 2, ...L$ for $k = 1$, and $l = 1, 2, ...L$ for $k = 2, ..., K$. Consequently, the relative phases $\phi_{l,k}(t)$ are updated according to

$$\dot{\phi}_{l,k}(t) = g_{\phi,l,k} \cdot y_{\mathrm{s},l,k}(t), \tag{34}$$

where the design parameters $g_{\phi,l,k}$ are all positive. As a result, the linear dynamics of the relative phase loops are given by

$$\dot{\phi}_{l,k}(t) = -g_{\phi,l,k} \cdot m_{\mathrm{d},l,k} \cdot \left[\phi_{l,k}(t) - \phi^0_{l,k}\right], \tag{35}$$

and the parameter estimates $\phi_{l,k}(t)$ converge to their ideal values as first-order systems with poles at $s = -g_{\phi,l,k} m_{\mathrm{d},l,k}$. As in the frequency loop, the dynamics of the relative phase loops are determined not only by the design parameters $g_{\phi,l,k}$, but also by the amplitudes $m_{\mathrm{d},l,k}$. Some prior knowledge about the values of $m_{\mathrm{d},l,k}$ is useful in the choice of the design parameters $g_{\phi,l,k}$, so that the poles $s = -g_{\phi,l,k} m_{\mathrm{d},l,k}$ are placed at desirable locations.

3.4 Effect of Output Disturbances

For overdetermined systems, the sensor signals $e_i(t)$ also contain the non-zero output disturbances $r_i(t)$, for $i = 1, 2, ..., I$. The multiplication of the disturbance vector $\mathbf{r}(t)$ by $\cos\left[\alpha_{l,k}(t)\right]$and $-\sin\left[\alpha_{l,k}(t)\right]$leads to

$$\begin{bmatrix} \mathbf{r}(t)\cos\left[\alpha_{l,k}(t)\right] \\ -\mathbf{r}(t)\sin\left[\alpha_{l,k}(t)\right] \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_k(t)\cos\left[\alpha_{l,k}(t)\right] \\ -\mathbf{Q}_k(t)\sin\left[\alpha_{l,k}(t)\right] \end{bmatrix} \left[-\mathbf{G}_k(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T + \mathbf{I}_{2I\times 2I}\right] \boldsymbol{\pi}_k. \tag{36}$$

We define $\delta_\alpha(t)$ such that $\alpha_{l,k}(t) = k\alpha(t) + \delta_\alpha(t)$. Neglecting high-frequency terms, as was done in the analysis of Section 3.2, (36) can be approximated to

$$\begin{bmatrix} \mathbf{r}(t)\cos[\alpha_{l,k}(t)] \\ -\mathbf{r}(t)\sin[\alpha_{l,k}(t)] \end{bmatrix} \simeq \frac{1}{2}\mathbf{T}[\delta_\alpha(t)]\left[-\mathbf{G}_k(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T + \mathbf{I}_{2I\times 2I}\right]\boldsymbol{\pi}_k, \tag{37}$$

where the matrix

$$\mathbf{T}[\delta_\alpha(t)] = \begin{bmatrix} \mathbf{I}_{I\times I}\cos[\delta_\alpha(t)] & \mathbf{I}_{I\times I}\sin[\delta_\alpha(t)] \\ -\mathbf{I}_{I\times I}\sin[\delta_\alpha(t)] & \mathbf{I}_{I\times I}\cos[\delta_\alpha(t)] \end{bmatrix}. \tag{38}$$

The effect of the disturbances $\mathbf{r}(t)$ on $y_{\mathrm{c},l,k}(t)$ and $y_{\mathrm{s},l,k}(t)$ is represented by the additive term

$$\mathbf{H}_k\begin{bmatrix} \mathbf{r}(t)\cos[\alpha_{l,k}(t)] \\ -\mathbf{r}(t)\sin[\alpha_{l,k}(t)] \end{bmatrix} = \frac{1}{2}(\hat{\mathbf{G}}_k^T\hat{\mathbf{G}}_k)^{-1}\hat{\mathbf{G}}_k^T\mathbf{T}[\delta_\alpha(t)]\left[-\mathbf{G}_k(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T + \mathbf{I}_{2I\times 2I}\right]\boldsymbol{\pi}_k. \tag{39}$$

If $\hat{\mathbf{G}}_k = \mathbf{G}_k$, since $\mathbf{G}_k^T\mathbf{T}[\delta_\alpha(t)] = \mathbf{S}[\delta_\alpha(t)]\,\mathbf{G}_k^T$, where the matrix

$$\mathbf{S}[\delta_\alpha(t)] = \begin{bmatrix} \mathbf{I}_{L\times L}\cos[\delta_\alpha(t)] & \mathbf{I}_{L\times L}\sin[\delta_\alpha(t)] \\ -\mathbf{I}_{L\times L}\sin[\delta_\alpha(t)] & \mathbf{I}_{L\times L}\cos[\delta_\alpha(t)] \end{bmatrix}, \tag{40}$$

we have

$$\begin{aligned}
& (\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T\mathbf{T}[\delta_\alpha(t)]\left[-\mathbf{G}_k(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T + \mathbf{I}_{2I\times 2I}\right] \\
= \; & -(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{S}[\delta_\alpha(t)]\,\mathbf{G}_k^T\mathbf{G}_k(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T + (\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T\mathbf{T}[\delta_\alpha(t)] \\
= \; & -(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{S}[\delta_\alpha(t)]\,\mathbf{G}_k^T + (\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T\mathbf{T}[\delta_\alpha(t)] \\
= \; & -(\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T\mathbf{T}[\delta_\alpha(t)] + (\mathbf{G}_k^T\mathbf{G}_k)^{-1}\mathbf{G}_k^T\mathbf{T}[\delta_\alpha(t)] \\
= \; & \mathbf{0}.
\end{aligned} \tag{41}$$

In this case, we have

$$\mathbf{H}_k\begin{bmatrix} \mathbf{r}(t)\cos[\alpha_{l,k}(t)] \\ -\mathbf{r}(t)\sin[\alpha_{l,k}(t)] \end{bmatrix} = \mathbf{0}. \tag{42}$$

In other words, the output disturbances $\mathbf{r}(t)$ for overdetermined systems do not affect the convergence of the adaptive parameters within the assumptions of the approximate analysis of the algorithm.

3.5 Computational Issues

Implementation of the algorithm requires the pseudo-inverse of the non-square matrix $\hat{\mathbf{G}}_k$, which may be computed using

$$(\hat{\mathbf{G}}_k^T \hat{\mathbf{G}}_k)^{-1} \hat{\mathbf{G}}_k^T = 4 \begin{bmatrix} (\hat{\mathbf{D}}_k + \hat{\mathbf{E}}_k \mathbf{D}_k^{-1} \hat{\mathbf{E}}_k)^{-1} & (\hat{\mathbf{D}}_k + \hat{\mathbf{E}}_k \mathbf{D}_k^{-1} \hat{\mathbf{E}}_k)^{-1} \hat{\mathbf{E}}_k \mathbf{D}_k^{-1} \\ -(\hat{\mathbf{D}}_k + \hat{\mathbf{E}}_k \mathbf{D}_k^{-1} \hat{\mathbf{E}}_k)^{-1} \hat{\mathbf{E}}_k \mathbf{D}_k^{-1} & (\hat{\mathbf{D}}_k + \hat{\mathbf{E}}_k \mathbf{D}_k^{-1} \hat{\mathbf{E}}_k)^{-1} \end{bmatrix} \hat{\mathbf{G}}_k^T \,, \quad (43)$$

with

$$\hat{\mathbf{D}}_k = \left(\hat{\mathbf{P}}_k^{\mathrm{Re}}\right)^T \hat{\mathbf{P}}_k^{\mathrm{Re}} + \left(\hat{\mathbf{P}}_k^{\mathrm{Im}}\right)^T \hat{\mathbf{P}}_k^{\mathrm{Im}} = \hat{\mathbf{D}}_k^T, \quad (44)$$

and

$$\hat{\mathbf{E}}_k = \left(\hat{\mathbf{P}}_k^{\mathrm{Re}}\right)^T \hat{\mathbf{P}}_k^{\mathrm{Im}} - \left(\hat{\mathbf{P}}_k^{\mathrm{Im}}\right)^T \hat{\mathbf{P}}_k^{\mathrm{Re}} = -\hat{\mathbf{E}}_k^T. \quad (45)$$

Note that the inverse is well-defined under the assumption made earlier that the plant frequency response matrix has full row rank at the frequency of each harmonic.

In a real-time implementation, the approximate inverse

$$(\hat{\mathbf{G}}_k^T \hat{\mathbf{G}}_k)^{-1} \hat{\mathbf{G}}_k^T \simeq 4 \begin{bmatrix} \hat{\mathbf{D}}_k & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{D}}_k \end{bmatrix}^{-1} \hat{\mathbf{G}}_k^T \quad (46)$$

may be used. The inverse is exact if $\hat{\mathbf{E}}_k = \mathbf{0}$, or $\left(\hat{\mathbf{P}}_k^{\mathrm{Re}}\right)^T \hat{\mathbf{P}}_k^{\mathrm{Im}} = \left(\hat{\mathbf{P}}_k^{\mathrm{Im}}\right)^T \hat{\mathbf{P}}_k^{\mathrm{Re}}$. This property is satisfied in particular if, for a 2×2 system, $\hat{P}_{11}(jk\omega) = \hat{P}_{22}(jk\omega)$, and $\hat{P}_{12}(jk\omega) = \hat{P}_{21}(jk\omega)$. These conditions may be viewed as "symmetry" conditions, in the sense that the behavior of the system is the same if the order of the inputs and outputs are permuted. In the experiments, the geometric disposition of the speakers and microphones was symmetric, and the conditions were verified to be approximately true.

4. Active Noise Control Application

The multivariable algorithm for rejection of periodic disturbances was applied to the reduction of acoustic noise in a confined environment. The control strategy was implemented on an experimental active noise control system developed at the University of Utah. Although the algorithm is formulated in the analog domain, it can be easily converted into the digital domain by approximation of derivatives (or any other transformation technique as is found in [11]). The algorithm was coded in the assembly language of Motorola's DSP96002 32-bit floating-point digital signal processor hosted in a PC. The sampling rate was set at 8 kHz. Fig. 6 represents the two-channel active noise control system. A single bookshelf speaker with a 4-inch low-frequency driver generated the periodic signal constituting the noise source. The signals were collected by two microphones separated by about 2.7 ft. These signals were passed through anti-aliasing filters and sampled by self-calibrating 16-bit analog-to-digital converters before being sent to the DSP system. The controller output signals were sent to two noise cancelling speakers positioned symmetrically with respect to the microphones. The whole system was set in a horizontal plane at about 2 ft in height.

Figure 6: Diagram of the ANC testbed.

The adaptive algorithm requires knowledge of the frequency response matrix $\mathbf{P}(j\omega)$ of the plant. Assuming that the characteristics of $\mathbf{P}(j\omega)$ are time-invariant but unknown, measurements can be used to estimate $\mathbf{P}(j\omega)$ during an initial training stage. At the end of the training interval, the estimated model $\hat{\mathbf{P}}(j\omega)$ is fixed and incorporated in the algorithm. The frequency response at a given frequency ω_0 was determined by the *empirical transfer function estimate* (ETFE, [12]) method. Let the first input (produced by control speaker #1) be a pure sinusoid $\cos(\omega_0 n)$ and the second input (produced by control speaker #2) be zero. $\hat{\mathbf{P}}_{1*}(j\omega_0)$, defined to be the first column of $\hat{\mathbf{P}}(j\omega_0)$, may be obtained through

$$\begin{aligned}\mathrm{Re}\left(\hat{\mathbf{P}}_{1*}\right) &= \frac{2}{N}\sum_{n=1}^{N}\mathbf{E}(n)\cos(\omega_0 n),\\ \mathrm{Im}\left(\hat{\mathbf{P}}_{1*}\right) &= -\frac{2}{N}\sum_{n=1}^{N}\mathbf{E}(n)\sin(\omega_0 n),\end{aligned}\tag{47}$$

where $\mathbf{E}(n)$ is the vector of plant outputs, and $N = k\pi/\omega_0$ with $k = 1, 2, 3.......$

The second column of $\hat{\mathbf{P}}(j\omega_0)$ may be obtained similarly. In the implementation of the algorithm, $I = L = 2$, and it was assumed that $P_{11}(j\omega) = P_{22}(j\omega)$, and $P_{12}(j\omega) = P_{21}(j\omega)$. This assumption need not be made in general, but the complexity of the code is reduced if the assumption is used. The real and imaginary parts of the frequency response were obtained at 64 different frequencies, spaced between 90 Hz and 375 Hz, and the results were saved in a look-up table. In real-time operation, the frequency response at the estimated frequency was obtained by linearly interpolating the look-up table, and the matrices $\hat{\mathbf{G}}_k$ were adjusted continuously as functions of the frequency estimate.

Figs. 7-10 show the estimates of the plant frequency responses. Fig. 7 and Fig. 8 are the magnitude response and the phase response of $P_{11}(j\omega)$, respectively, while Fig. 9 and Fig. 10 are the corresponding responses of $P_{21}(j\omega)$. The phase responses mostly consist of the linear phase associated with the delay due to sound propagation from the speakers to the microphones. The magnitude responses show a significant number of peaks and valleys, which are due to acoustic resonances and typical for the acoustic plant response in an enclosure.

Figure 7: Magnitude response of $P_{11}(s)$.

Figure 8: Unwrapped phase response of $P_{11}(s)$ (rad.).

Figure 9: Magnitude response of $P_{21}(s)$.

Figure 10: Unwrapped phase response of $P_{21}(s)$ (rad.).

5. Experimental Results of Active Noise Control

5.1 Fixed Frequency Experiments

For all the experiments in this paper, the noise contains a fundamental and a 2nd harmonic. In this set of results, the fundamental frequency of the noise was fixed at 130 Hz. The adaptation gains for the amplitude loops were $g_{\mathrm{m}} = 4.0$ for all components of the signals, leading to closed-loop poles at -4 rad/s for the linearized systems of the amplitude loops. The design parameters for the frequency loop were set to $K_{\mathrm{f}} = 0.1$ and $g_{\omega} = 800$. The design parameters for the relative phase loops were set to $g_{\phi,1,2} = 60$, $g_{\phi,2,1} = 60$, and $g_{\phi,2,2} = 250$.

Fig. 11 shows the signals obtained from error microphones #1 and #2. For the purpose of evaluation, the control system was switched on at 1 second into this recording. The figure shows that the algorithm, once engaged, reduced both noise signals considerably within one second. In steady-state, the fundamental components and the harmonics of the two noise signals were completely eliminated.

Figure 11: Microphone signals.

Fig. 12 shows the amplitudes of the fundamental component and of the 2nd harmonic for control signals #1 and #2. The parameters converged to the steady-state values in about 1 second, consistently with the magnitude loop poles of -4 rad/s. The poles for the relative phase loops were in the vicinity of -8 rad/s, with a settling time of about 0.5 seconds. The dynamics of the linear approximation of the frequency loop were determined by the roots of $s^2 + 16s + 160 = 0$, i.e., $-8 \pm j4\sqrt{6}$ rad/s. The corresponding settling time was also 0.5 seconds. Fig. 13 shows the frequency estimate, which converged to its nominal value in about 0.5 seconds. Fig. 14 shows the relative phases with an approximate convergence time of 1 second. This value is the sum of the convergence time of the frequency loop and the convergence time of the relative phase loops predicted from the linear approximation.

Figure 12: Amplitudes of the control signals.

5.2 Varying Frequency Experiments

In the second case, the fundamental frequency of the noise was increased linearly from 130 Hz to 150 Hz in 10 seconds. The results of this section show that disturbances with varying frequencies can also be reduced significantly. Due to the acoustic properties of the room, the amplitudes of the noise signals at the two microphone locations changed slowly, although the amplitudes of the components of the real noise source were constant. The adaptation gains for the update of the control signal amplitudes were increased to $g_{\mathrm{m}} = 8.0$ for all components of the signals, in order to improve tracking. The design parameters for the frequency and relative phase loops were set to $K_{\mathrm{f}} = 0.04$, $g_\omega = 1200$, $g_{\phi,1,2} = 40$, $g_{\phi,2,1} = 60$, and $g_{\phi,2,2} = 100$.

Figure 13: Frequency estimate of the control signals.

Figure 14: Relative phases of the control signals.

Figs. 15 and 16 show noise signals #1 and #2 at the microphone locations without control (upper part) and the corresponding residual error signals when the control is engaged (lower part). The frequency tracking performance and the frequency tracking error are shown in Fig. 17. The frequency tracking error is about 0.2 Hz. Fig. 18 shows the amplitudes of the fundamental component and of the 2nd harmonic for control signal #1 and #2. Fig. 19 shows that the relative phase of the fundamental component of noise signal #2 increases linearly, with a rate of 0.08 Hz. This variation compensates partly for the frequency tracking delay. The performance of the relative phase estimation for the 2nd harmonic is not as good, because the amplitudes of the 2nd harmonics are small and change fast, as shown in Fig. 18. Rapid changes of the phase estimates by 180° are associated with small values of the corresponding amplitude parameters of the noise signals, which may be attributed to (approximate) zeros of transmission from the noise source to the microphones.

Figure 15: Noise signal #1 with and without control.

Figs. 20 and 21 show the spectra of the noise signals (solid lines) and the corresponding residual error signals under control (dashed lines), obtained from microphones #1 and #2 (respectively). The power spectral density of the signals are estimated using Welch's averaged periodogram method with non-overlapping Hanning window with a length of 800 samples (using the function spectrum.m in Matlab). The signals after 1.0 second were used in the spectral analysis in order to demonstrate

Figure 16: Noise signal #2 with and without control.

Figure 17: Frequency tracking and the error.

Figure 18: Amplitude parameter estimates.

Figure 19: Relative phase parameter estimates.

the performance of the control system in steady-state. The noise signals shown in the spectra have significant spectral content from 132 Hz to 150 Hz and from 264 Hz to 300 Hz. For error signal #1, the contribution of the fundamental component was reduced by approximately 30 dB, and the 2nd harmonic by 10 dB (on average). For error signal #2, the fundamental component was reduced by approximately 34 dB, and the 2nd harmonic by 15 dB. The spectral peaks of the narrowband noise signals are still visible against the background noise, and the noise reduction of the 2nd harmonic is not as good as that of the fundamental. However, a considerable overall reduction of the noise is achieved despite rapid changes in signal characteristics.

Figure 20: Spectra of noise #1 and its residual.

6. Conclusions

An algorithm was proposed for the active control of periodic disturbances of unknown frequency at multiple sensor locations. The nominal solution was first discussed under a full-rank assumption, and it was shown that disturbances could be canceled exactly for square systems, while residuals existed for overdetermined systems. The disturbances were then reformulated as the superposition of disturbance sources at the plant input and disturbances at the plant output, which are equivalent to the residuals.

The proposed scheme is a complex nonlinear dynamic system. However, various approximations

Figure 21: Spectra of noise #2 and its residual.

and linearization around the nominal solution enabled a relatively straightforward linear design. Within the assumptions, it was found that the output disturbances had no effect on the convergence of the scheme. The analysis also gave useful information about the dynamic behavior of the system and the selection of the design parameters. An interesting feature of the algorithm is that the frequencies of all the components of the control signals are tied to a single frequency estimate, reflecting the assumptions made about the source of the disturbances. The multivariable control algorithm was applied to the reduction of periodic acoustic noise in a small room. Experimental results showed that the proposed algorithm was able to significantly reduce acoustic noises with constant or time-varying signal parameters.

7. References

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