

# Study of the Stability of a Direct Stator Current Controller for a Doubly-Fed Induction Machine using the Complex Hurwitz test

A. Dòria-Cerezo, M. Bodson, C. Batlle and R. Ortega

## Abstract

In this paper, a new control scheme is presented for the doubly-fed induction machine with specific applications to renewable energy (wind farms in particular). The proposed control algorithm offers the advantages of proven stability and remarkable simplicity. In contrast with the classical vector control method, where the doubly-fed induction machine is represented in a stator-flux oriented frame, a model with orientation of the stator voltage is adopted. This approach allows for a decomposition of the active and reactive powers on the stator side and their regulation on the rotor side. A main contribution of the paper is the use of a Hurwitz test for polynomials with complex coefficients that has had little prior application in control theory. This results in a proof that a PI control regulating the stator currents ensures global stability for a feedback-linearized doubly-fed induction machine. The specific condition that the PI gains must satisfy is derived as a simple inequality. The PI controller has a particular structure which directly relates the d-component of the rotor voltages to the q-component of the stator currents and vice-versa. The feedback linearization stage only uses the direct measurement of the rotor and stator currents and is thus easily implementable. Furthermore, it is also shown that the PI controller (without the feedback linearization terms) is also stable for a large range of control gains and does not require the knowledge of the machine parameters. Finally, the control system is validated in simulations and in experiments.

## I. INTRODUCTION

Doubly-fed induction machines (DFIM) have become very popular, especially in the field of renewable energy as hybrid engines or high performance storage systems [1][2][3][4] and for wind turbines [5][6][7]. The attractiveness of the DFIM stems primarily from its ability to handle large speed variations around the synchronous speed. Another advantage is that the power electronic equipment that controls the machine only has to handle a fraction of the total power (which is directly related to the operating speed [2]), reducing losses and the cost of the power electronic converter.

In this paper, a typical connection of the DFIM is considered. In this case, the stator is directly connected to the power grid, while the machine is controlled through the rotor voltages. A back-to-back (B2B) converter, consisting of an AC-DC rectifier and a DC-AC inverter stage, is used for generating the rotor voltages. For generation, the control goals for a DFIM are usually the active and reactive powers delivered to the grid. For drive applications, the DFIM control is composed of an inner current control loop and an external (and considerably slower) mechanical loop (see in [8] a counter example with a unique control loop). This paper only focusses on the electrical loop, and assumes that, for driving applications or power management of the whole DFIM and B2B, an outer mechanical loop (in terms of torque or speed) or a power supervisor (such as Maximum

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Power Point Tracking for wind generators), provides the active power reference for the electrical controller. The speed is also assumed to vary slower than the electrical variables.

Most DFIM controllers proposed in the literature are based on vector control and decoupling [9][6][10][11], also known as PQ control. The methodology is derived from the description of the electrical part of the DFIM in a stator flux-oriented reference frame which allows the decoupling of the active and reactive powers of the stator side and their independent control through the rotor currents. Moreover, some of the controllers that use the stator-flux oriented frame assume a fast stator flux dynamics which, in steady state, is used to reduce the order of the system. To achieve the stator flux orientation, the flux angle must be estimated and several operations implemented. In contrast to the stator flux-oriented frame, a model with orientation along the stator voltage vector can be considered [12]. Assuming an ideal power grid, control of the stator currents in this reference frame directly translates into the control of the active and reactive powers absorbed or delivered by the machine. Control algorithms are based on an inner rotor current loop, and an external PQ loop control. See recent examples in [13][14].

Examples on direct control of the stator currents using the stator voltage reference frame can be found in [1][2][8][15]. In these cases the control scheme contains only one loop from the state (or output) measurement to the controller. In [1], the control algorithm was designed disregarding the rotor dynamics and it results in a state-feedback controller plus a PI action. However, the controller makes the whole closed loop system marginally stable, see discussion in [16]. The static state-feedback control laws proposed in [2] and [8] were obtained using passivity-based techniques and ensure global asymptotic stability of the DFIM (including the mechanical part), but they are very sensitive to the parameter variations. Also, advanced nonlinear methods have been applied to the DFIM. In [17] the flatness of the model has been exploited to reduce the power losses, and a controller based on the backstepping approach is proposed. Finally, in [15] a nonlinear technique based on the Lyapunov function was used to design a direct current algorithm with a feedforward action that allows to track stator current references. This scheme requires the reconstruction of the rotor flux references and the exact knowledge of several DFIM parameters.

The main contribution of this paper, which is based on [18], is the proof that a linear PI control for regulating the stator currents does not allow to place the closed loop poles arbitrarily, but ensures stability for a large range of PI gain values. In comparison with the PQ stator voltage oriented control approaches, the power regulation is achieved with one PI loop, instead of the composition of an outer power regulator and an inner rotor current loop control. Moreover, the direct PI controller studied in this paper is considerably simpler in comparison with other direct stator current schemes. Furthermore, global stability can be guaranteed if a feedback linearizing term is added. This feedthrough term only uses the direct measurement of the stator and rotor currents (both accessible for a DFIM), instead of the flux estimation required in the stator flux oriented methods.

The stability proofs are based on a little-known Hurwitz test for complex polynomials [19]. This method allows to significantly reduce the complexity of obtaining the stability conditions by reducing the 6th order characteristic polynomial with real coefficients to a cubic polynomial with complex coefficients. Interestingly, while a Routh-Hurwitz test for the 6th order polynomial of [20] was found to be intractable, application of its version with complex polynomials yields a simple stability test, requiring that a single quadratic inequality be satisfied by the PI gains. The method proposed in this paper is also applicable to other control problems with certain symmetry properties. In [21], the Hurwitz test was used to find analytic conditions for spontaneous self-excitation in induction generators. The same test was applied to the algorithm presented in [15], see [18].

## II. BACKGROUND: THE COMPLEX HURWITZ TEST

The extension of the well-known Routh-Hurwitz criterion to polynomials with complex coefficients is an old result of the literature [19], possibly not well-known due to the lack of relevant applications. The main result presented in that paper is summarized by the following Theorem.

*Theorem 1:* The polynomial  $P(s) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \dots + \alpha_n$ , where  $\alpha_k = a_k + jb_k$  and  $k = 1, 2, \dots, n$ , has all its zeros in the half-plane  $\Re(s) < 0$  if and only if the determinants,  $\Delta_1 \dots \Delta_k$ ,

$$\Delta_1 = a_1$$

$$\Delta_k = \begin{vmatrix} a_1 & a_3 & a_5 & \dots & a_{2k-1} & -b_2 & -b_4 & \dots & -b_{2k-2} \\ 1 & a_2 & a_4 & \dots & a_{2k-2} & -b_1 & -b_3 & \dots & -b_{2k-3} \\ \vdots & & & \ddots & \vdots & & & \ddots & \vdots \\ 0 & & & \dots & a_k & 0 & & \dots & -b_{k-1} \\ 0 & b_2 & b_4 & \dots & b_{2k-2} & a_1 & a_3 & \dots & a_{2k-3} \\ 0 & b_1 & b_3 & \dots & b_{2k-3} & 1 & a_2 & \dots & a_{2k-4} \\ \vdots & & & \ddots & \vdots & & & \ddots & \vdots \\ 0 & & & \dots & b_k & 0 & & \dots & a_{k-1} \end{vmatrix}$$

for  $k = 2, 3, \dots, n$  and  $a_r = b_r = 0$  for  $r > n$ , are all positive.

*Proof:* See Theorem 3.2 of [19]. ■

Based on the previous Theorem, the particular case of a cubic polynomial with complex coefficients can be derived.

*Lemma 1:* Assuming that  $a_0$  is real and positive, the roots of a third-order polynomial with complex coefficients

$$P(s) = a_0 s^3 + (a_1 + jb_1)s^2 + (a_2 + jb_2)s + a_3 + jb_3 \quad (1)$$

are in the open left-half plane if and only if  $\Delta_1 > 0$ ,  $\Delta_2 > 0$  and  $\Delta_3 > 0$ , where

$$\Delta_1 = a_1, \quad (2)$$

$$\Delta_2 = \begin{vmatrix} a_1 & a_3 & -b_2 \\ a_0 & a_2 & -b_1 \\ 0 & b_2 & a_1 \end{vmatrix}, \quad (3)$$

$$\Delta_3 = \begin{vmatrix} a_1 & a_3 & 0 & -b_2 & 0 \\ a_0 & a_2 & 0 & -b_1 & -b_3 \\ 0 & a_1 & a_3 & 0 & -b_2 \\ 0 & b_2 & 0 & a_1 & a_3 \\ 0 & b_1 & b_3 & a_0 & a_2 \end{vmatrix}. \quad (4)$$

We will show in Sections IV and V how the complex Hurwitz test can be used to prove stability of a DFIM control law in an elegant and simple manner.

### III. MODEL OF THE DOUBLY-FED INDUCTION MACHINE

The model comes from the three phase dynamical equations of a DFIM, assuming that the machine is symmetric (all windings are identical), the stator-rotor mutual inductances are sinusoidal functions of the rotor angle [22][23], and the three phase system is balanced. These assumptions enable the use of transformations, which greatly simplify the control problem. The basic transformation (also known as Blondel–Park transformation) is widely used in the study of electric machines [23]. This mathematical transformation is used to decouple one of the (balanced) phases, to refer all variables to a common reference frame, and to obtain state-space models whose parameters are independent of the relative angle between rotor and stator.

Similarly to [1] or [2], a transformation to a synchronous frame rotating at the constant frequency of the stator voltage of the grid is proposed. Following standard convention, all electrical (two–dimensional vector) signals are partitioned into their so–called  $d$  and  $q$  components. This yields the electrical equations

$$L_s \frac{di_{sd}}{dt} + L_{sr} \frac{di_{rd}}{dt} = -R_s i_{sd} + \omega_s L_s i_{sq} + \omega_s L_{sr} i_{rq} + v_{sd} \quad (5)$$

$$L_s \frac{di_{sq}}{dt} + L_{sr} \frac{di_{rq}}{dt} = -\omega_s L_s i_{sd} - R_s i_{sq} - \omega_s L_{sr} i_{rd} + v_{sq} \quad (6)$$

$$L_{sr} \frac{di_{sd}}{dt} + L_r \frac{di_{rd}}{dt} = (\omega_s - \omega) L_{sr} i_{sq} - R_r i_{rd} + (\omega_s - \omega) L_r i_{rq} + v_{rd} \quad (7)$$

$$L_{sr} \frac{di_{sq}}{dt} + L_r \frac{di_{rq}}{dt} = -(\omega_s - \omega) L_{sr} i_{sd} - (\omega_s - \omega) L_r i_{rd} - R_r i_{rq} + v_{rq} \quad (8)$$

where  $i_{sd}, i_{sq}, i_{rd}, i_{rq}$  are stator and rotor currents, respectively,  $v_{sd}, v_{sq}$  are the stator voltages,  $v_{rd}, v_{rq}$  are the rotor voltages (and play the role of the control inputs),  $\omega$  is the mechanical speed, and  $\omega_s$  is the stator frequency.  $R_s, R_r$  are the stator and rotor resistances,  $L_s, L_r$  and  $L_{sr}$  are the stator and rotor self–inductances and mutual inductance, with  $L_s L_r > L_{sr}^2$ .

The mechanical equation is given by

$$J \frac{d\omega}{dt} = L_{sr} (i_{sq} i_{rd} - i_{sd} i_{rq}) - B_r \omega - \tau_L \quad (9)$$

where  $J$  is the inertia,  $B_r$  is the friction coefficient, and  $\tau_L$  is an external constant torque.

The use of the stator-voltage oriented synchronous frame,  $v_{sd} = V_s$  and  $v_{sq} = 0$  (with  $V_s$  the amplitude of the three-phase stator voltage), allows us to express the stator active and reactive powers in terms of  $i_{sd}$  and  $i_{sq}$ , respectively, so that,

$$P_s = v_{sd} i_{sd} + v_{sq} i_{sq} \quad (10)$$

$$Q_s = v_{sq} i_{sd} - v_{sd} i_{sq} \quad (11)$$

simplify to

$$P_s = V_s i_{sd} \quad (12)$$

$$Q_s = -V_s i_{sq}. \quad (13)$$

In particular, assigning a desired value  $i_{sq}^*$  allows one to regulate the power factor of the stator side of the machine, while  $i_{sd}^*$  can be used to control the active power (delivered or consumed) by the DFIM. In a drive application,  $i_{sd}^*$  is fixed as a

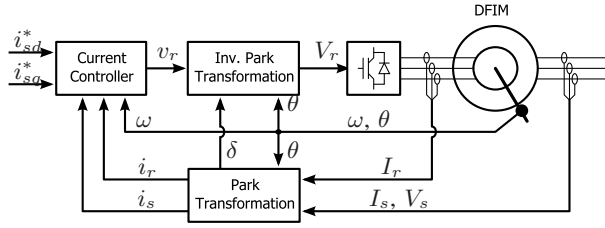


Fig. 1. Proposed feedback-linearizing control scheme for a DFIM.

desired value to achieve the target torque in the presence of voltage constraints. This paper concentrates only on the problem of robust regulation of  $i_s$  to its desired value.

#### IV. FEEDBACK-LINEARIZING CURRENT CONTROLLER

The proposed control scheme is presented in Figure 1. It is composed of the current control block and the well-known Blondel-Park transformation to recover the dq-measurements of the currents and the stator voltages, and its inverse to generate the three-phase rotor voltages from the computed  $v_r$  values (in the dq-framework). Thanks to the use of the stator voltage reference (already used in [1] or [12] among others) the proposed algorithm is simpler than the classic stator-flux oriented control [6], where the stator flux estimation (or reconstruction) is required for the reference frame orientation. Furthermore, the stator voltage oriented framework allows us to directly use the stator currents  $i_s$  to regulate the active and reactive stator power, see equations (12) and (13). This fact also simplifies the algorithm compared with the standard approach.

The control algorithm can be also used as the inner-current loop for drive applications. Fixing  $i_{sq}$  for the reactive power compensation, a suitable  $i_{sd}$  can be obtained from a mechanical outer-loop for the torque, or speed regulation (from the mechanical dynamics (9)).

##### A. Feedback-linearizing current controller

The proposed controller consists of a partial feedback linearizing feedback stage

$$v_{rd} = -(\omega_s - \omega)L_{sr}i_{sq} + R_r i_{rd} - (\omega_s - \omega)L_r i_{rq} + u_d \quad (14)$$

$$v_{rq} = (\omega_s - \omega)L_{sr}i_{sd} + (\omega_s - \omega)L_r i_{rd} + R_r i_{rq} + u_q \quad (15)$$

and a PI action

$$u_d = -k_P(i_{sq}^* - i_{sq}) - k_I \int_{t_0}^t (i_{sq}^* - i_{sq}) d\tau \quad (16)$$

$$u_q = k_P(i_{sd}^* - i_{sd}) + k_I \int_{t_0}^t (i_{sd}^* - i_{sd}) d\tau \quad (17)$$

with the scalar proportional and integral gains  $k_P$  and  $k_I$ , respectively.

*Remark 1:* The first three terms in (14) and (15) exactly cancel the first three terms in (7) and (8), respectively, feedback-linearizing the system and transforming the rotor equations to  $L_{sr} \frac{di_{sd}}{dt} + L_r \frac{di_{rd}}{dt} = u_d$  and  $L_{sr} \frac{di_{sq}}{dt} + L_r \frac{di_{rq}}{dt} = u_q$ .

*Remark 2:* In contrast to [1], the PI controller is defined relating the d-rotor voltage component with (minus) the error of the q-stator current (16), and the q-rotor voltage with (plus) the error of the d-stator current (17). This fact turns out to be critical for the stability analysis and solves the marginally stability problem of [1], see [16]. As explained in [24], this structure was suggested by applying passivity-based nonlinear control techniques which, precisely, takes into account the original DFIM dynamics where the dq stator currents appear rotated in the equations of the rotor fluxes (7)-(8). In [15] the cross-terms were also required to prove global stability, see Fig. 3 in [15].

Substituting (14)-(15) in (7)-(8), a linear closed-loop system is obtained. Applying the Laplace transform, stability of the system is determined by a 6th order characteristic polynomial, which was analyzed in [20] but without reaching complete analytic conditions for stability. Interestingly, this sixth order characteristic polynomial with real coefficients can be reduced into a 3rd order polynomial with complex coefficients for which complete analysis is possible.

Defining  $\mathcal{I}_s(s) = I_{sd}(s) + jI_{sq}(s)$ ,  $\mathcal{I}_r(s) = I_{rd}(s) + jI_{rq}(s)$ ,  $\mathcal{V}_s(s) = V_{sd}(s) + jV_{sq}(s)$ ,  $\mathcal{V}_r(s) = V_{rd}(s) + jV_{rq}(s)$ , the closed loop system can be written as

$$\mathcal{A}(s) \begin{pmatrix} \mathcal{I}_s(s) \\ \mathcal{I}_r(s) \\ \mathcal{V}_r(s) \end{pmatrix} = \begin{pmatrix} \mathcal{V}_s(s) \\ 0 \\ j(k_P s + k_I)\mathcal{I}_s^*(s) \end{pmatrix} \quad (18)$$

where

$$\mathcal{A}(s) = \begin{pmatrix} L_s s + R_s + j\omega_s L_s & L_{sr} s + j\omega_s L_{sr} & 0 \\ L_{sr} s & L_r s & -1 \\ j(k_P s + k_I) & 0 & s \end{pmatrix}. \quad (19)$$

Notice that the polynomial  $\det \mathcal{A}(s)$  has 3 roots that are not required to be real or appear as complex pairs. The complex polynomial has the form

$$\det \mathcal{A}(s) = a_0 s^3 + (a_1 + jb_1)s^2 + (a_2 + jb_2)s + a_3 + jb_3 \quad (20)$$

where the parameter values are given in [18].

Now the stability of the closed loop system (5)-(8) with (14)-(15), can be analyzed with Lemma 1, where  $a_0 = \mu > 0$  ( $\mu = L_s L_r - L_{sr}^2 > 0$ , in all electrical machines) is fulfilled. Computing conditions (2)-(4) yields

$$\Delta_1 = L_r R_s \quad (21)$$

$$\Delta_2 = L_{sr} (k_P \omega_s L_r^2 R_s^2 + k_P k_I L_{sr} L_r R_s - 2k_I \omega_s \mu L_r R_s - k_I^2 \mu L_{sr}) \quad (22)$$

$$\Delta_3 = k_I \omega_s^3 L_{sr}^2 L_r R_s (k_P^2 L_{sr} L_r R_s - k_P k_I \mu L_{sr} - k_I \omega_s \mu^2). \quad (23)$$

The first condition is automatically fulfilled because  $L_r, R_s > 0$ . It can be shown (see[18]) that one must have  $k_I > 0$ , and the third condition is more restrictive than the second one. Consequently, the Hurwitz conditions reduce to  $\Delta_3 > 0$ . Notice that the stability condition does not depend on the mechanical speed. This stability condition has as asymptote the result presented in [20], which implies a generalization of the previous work.

*Proposition 1:* Consider the DFIM system (5)-(8) in closed-loop with the control law (14)-(15). If

$$0 < k_I < \frac{k_P^2 L_{sr} L_r R_s}{\mu (\mu \omega_s + k_P L_{sr})} \quad (24)$$

the closed-loop system is asymptotically stable.

*Proof:* See [18]. ■

Finally, it is worth mentioning that the degrees of freedom provided by the PI gains does not allow us to place the poles in arbitrary locations, which are depending on the DFIM parameters and, consequently, the convergence speed can be bounded by the machine parameters (see example in Figure 8). A qualitative design rule can be obtained numerically using a Root Locus Analysis. Moreover, for generation applications, it is crucial to have resistance  $R_s$  and  $R_r$  as low as possible which tends to have  $\Delta_1 \approx 0$ . This fact could affect the stability if there are delay effects in the feedback loop. The use of both “direct” and cross-terms of the stator current errors, as [15], is a possible solution to relax the influence of the  $R_s$  and  $R_r$  values.

### B. Effect of an incorrect rotor resistance estimation

As seen from (14)-(15), the feedback linearization term requires the knowledge of  $R_r$ ,  $L_s$  and  $L_{sr}$ , which are in general uncertain and time-varying parameters. In particular, due to thermal effects, the value of  $R_r$  is highly varying. In order to evaluate the effect of a possibly incorrect estimation of  $R_r$ , let us assume that one uses in (14)-(15) an estimated value  $\hat{R}_r$ , which can differ from the actual value of the rotor resistance. Then, defining  $\tilde{R}_r := R_r - \hat{R}_r$ , (19) slightly modifies to

$$\mathcal{A}_r(s) = \begin{pmatrix} L_s s + R_s + j\omega_s L_s & L_{sr} s + j\omega_s L_{sr} & 0 \\ L_{sr} s & L_r s + \tilde{R}_r & -1 \\ j(k_P s + k_I) & 0 & s \end{pmatrix}. \quad (25)$$

Applying again Lemma 1 to the new matrix  $\mathcal{A}_r(s)$ , it is possible to obtain a set of conditions given by  $\Delta_1, \Delta_2, \Delta_3 > 0$ . The determinant of  $\mathcal{A}_r(s)$  still has the form of (20), with the same parameters of [18] except for the following three:

$$a_1 = L_r R_s + L_s \tilde{R}_r \quad (26)$$

$$a_2 = k_P \omega_s L_{sr} + R_s \tilde{R}_r \quad (27)$$

$$b_2 = -k_I L_{sr} + \omega_s L_s \tilde{R}_r. \quad (28)$$

First, note that  $a_1 > 0$ , and consequently  $\Delta_1 > 0$ , if and only if  $\hat{R}_r < R_r + \frac{L_r}{L_s} R_s$ , which warns against an overestimation of  $R_r$ . Conditions  $\Delta_2 > 0$  and  $\Delta_3 > 0$  are quite more complicated,

$$\Delta_2 = c_1 k_I^2 + c_2 k_P k_I + c_3 k_I + c_4 k_P + c_5 \quad (29)$$

$$\Delta_3 = d_1 k_I^3 + d_2 k_P k_I^2 + d_3 k_I^2 + d_4 k_P^2 k_I + d_5 k_P k_I + d_6 k_I \quad (30)$$

where the parameters are detailed in Appendix A.

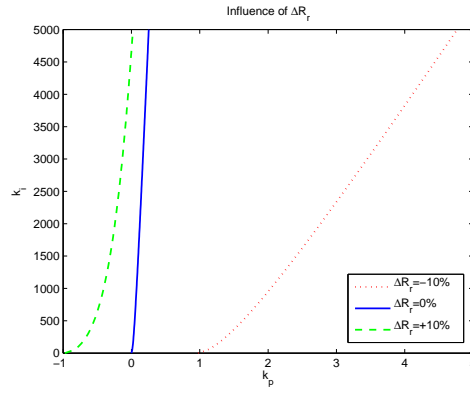


Fig. 2. Stability boundaries given by (29) and (30) for the proposed control law (14)-(15) with an uncertain value of  $R_r$ .

At this point, in order to simplify the stability conditions, it is possible to compute the positive slope asymptotes of (29) and (30), which turn out to be the same and given by

$$k_I = \frac{L_r R_s + L_s \tilde{R}_r}{\mu} k_P - \frac{\omega_s L_r R_s}{L_{sr}}. \quad (31)$$

Notice that for  $\tilde{R}_r = 0$ , this corresponds to the asymptote of (24), and the stability condition presented in [20] is recovered. Also, it is worth mentioning that negative values for  $\hat{R}_r$  will increase the slope of (31) which implies a larger stability region. However, this selection will affect the performance of the controller with a longer stabilization time.

Figure 2, shows a numerical example of the stability region for different values of  $\hat{R}_r$ . With the parameters of the machine described in Section VI, and considering a 10% error in the  $R_r$  estimation,  $\Delta_3 > 0$  is more restrictive than  $\Delta_2 > 0$ . In Figure 2, the  $\Delta_3 = 0$  for different errors in the rotor resistance estimation are shown. This numerical example shows that a positive error, *i.e.*  $\tilde{R}_r > 0$ , in the estimation of  $\hat{R}_r$  is preferred. In other words, overestimating the rotor resistance implies decreasing the stability region and, as pointed out before, can even destabilize the system. Moreover, for small values of  $\tilde{R}_r$ , a small stable region appears for  $k_P < 0$ . This can be easily seen from the asymptote (31), where  $\tilde{R}_r < 0$  implies a lower slope. This fact suggests taking  $\hat{R}_r = 0$  to have a larger stability region, and also simplify the control law. Then, asymptote (31) yields,

$$k_I = \frac{L_r R_s + L_s \tilde{R}_r}{\mu} k_P - \frac{\omega_s L_r R_s}{L_{sr}}. \quad (32)$$

## V. PI STATOR CURRENT CONTROLLER

The control law introduced in the previous section guarantees stability for a large range of the PI parameter values. However, in order to implement the control algorithm, it is necessary to know some machine parameters and both the stator and the rotor currents. In this section a simplification of the proposed controller (14)-(15) is analyzed that only keeps the PI action, *i.e.*,

$$v_{rd} = -k_P(i_{sq}^* - i_{sq}) - k_I \int_{t_0}^t (i_{sq}^* - i_{sq}) d\tau \quad (33)$$

$$v_{rq} = k_P(i_{sd}^* - i_{sd}) + k_I \int_{t_0}^t (i_{sd}^* - i_{sd}) d\tau. \quad (34)$$

For this scheme, shown in Figure 3, the rotor currents are not required, and only the stator currents need to be measured.



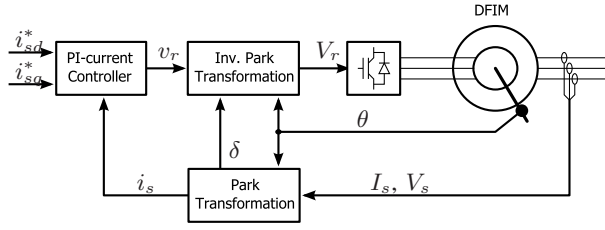


Fig. 3. Simplified control scheme for a DFIM.

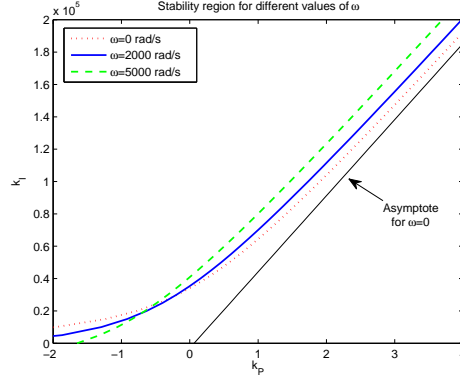


Fig. 4. Stability conditions (29), (30) for the control algorithm (33)-(34) for different values of  $\omega$ .

Notice that, in a torque controller application, the rotor currents could be required to set the  $i_d^*$  value in an outer control loop.

At this point, for the stability analysis, a constant mechanical speed is assumed. Using the same idea as before, the closed-loop dynamics can be written as (18) with  $\mathcal{A}(s)$  becoming

$$\mathcal{A}_{PI}(s) = \begin{pmatrix} L_s s + R_s + j\omega_s L_s & L_{sr} s + j\omega_s L_{sr} & 0 \\ L_{sr} s + j(\omega_s - \omega)L_{sr} & L_r s + R_r + j(\omega_s - \omega)L_r & -1 \\ j(k_P s + k_I) & 0 & s \end{pmatrix}. \quad (35)$$

The polynomial  $\det \mathcal{A}_{PI}(s)$  still has 3 roots, and the Hurwitz test described in Lemma 1 can be used. The determinant of (35) has the same form as (20) with the coefficients given in Appendix B. The stability of the closed loop system (5)-(8) with (33)-(34) can be analyzed by computing  $\Delta_1$ ,  $\Delta_2$ ,  $\Delta_3$ .  $\Delta_1 = a_1$  so that  $\Delta_1 > 0$ , while  $\Delta_2$  and  $\Delta_3$  are in the same form as (29) and (30), respectively, where the coefficients now take the values given in Appendix C.

To obtain an expression for the stability region becomes complicated. As a first result, conditions can be plotted for a numerical case. Using the machine parameters of Section VI, equations (29) and (30) are obtained for different values of the mechanical speed. Figure 4 shows the stability regions for the mechanical speed.

In order to bound the stability region, it is possible to find the asymptote of (29) and (30). Similarly to the case for an unknown rotor resistance presented in the previous section, the asymptotes for the  $\Delta_2 = 0$  and  $\Delta_3 = 0$  are equal and given by

$$k_I = \frac{L_r R_s + L_s R_r}{\mu} k_P + \frac{\omega L_s R_r - \omega_s (L_r R_s + L_s R_r)}{L_{sr}}. \quad (36)$$

Note that the slope of the stability boundary does not depend on the mechanical speed. As the worst case is when  $\omega = 0$ , the

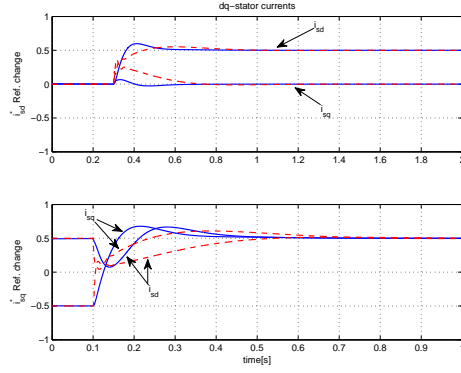


Fig. 5. Simulation results: stator currents under a change in the references (FL+PI: solid line, PI: dotted line). Top: from  $i_s^* = [0, 0]$ A to  $i_s^* = [0.5, 0]$ A. Bottom: from  $i_s^* = [0.5, -0.5]$ A to  $i_s^* = [0.5, 0.5]$ A.

stability for the PI controller proposed can be ensured by setting

$$k_I < \frac{L_r R_s + L_s R_r}{\mu} k_P - \frac{\omega_s (L_r R_s + L_s R_r)}{L_{sr}}. \quad (37)$$

## VI. SIMULATIONS

The proposed controllers were tested in numerical experiments using Matlab. The DFIM parameters are the same than ones used in Section VII. The DFIM is connected to a 380V and 50Hz power grid. Using a power preserving transformation and the stator-voltage oriented frame, this corresponds to a constant stator voltage vector  $v_s = [380, 0]$ V and  $\omega_s = 100\pi$ rad s<sup>-1</sup>.

A first simulation consists in a comparison between the two presented control schemes. For this test, the dq-model (5)-(8) is in a closed loop with the control schemes (14)-(15) and (33)-(34). The knowledge of all the parameters is assumed for the feedback linearizing control scheme and the control gains of the PI action are fixed at  $k_P = 0.5$ VA<sup>-1</sup>,  $k_I = 3$ VA<sup>-1</sup>s<sup>-1</sup> for the FL+PI controller and  $k_P = 5$ VA<sup>-1</sup>,  $k_I = 50$ VA<sup>-1</sup>s<sup>-1</sup> for the Direct PI controller. The mechanical speed is set to  $\omega = 325$ rad s<sup>-1</sup>. As shown in Figure 5, both algorithms perform in a similar way. This result suggests the use of the direct PI controller because it is easily implementable; rotor currents are not needed, and the knowledge of the machine parameters is not required. However, the stability of the control law (33)-(34) is based on the assumption of constant mechanical speed. In the simulation depicted in Figure 6, the current references are  $i_s^* = [0.5, 0]$ A and the mechanical speed is modified. When a sudden change of the velocity occurs, both d and q stator currents reach the desired values after a short time. Also, note that during the acceleration of the mechanical speed (from  $t = 0.5$ s to  $t = 1.5$ s), the d-stator current tracks the reference value but, in the Direct PI algorithm the q-stator current has some steady-state error. This suggests the proposed controller in Section IV-A for applications with mechanical speed variations.

The second test compares the proposed method (the so-called direct PI controller) with an existing PQ control which also uses the stator voltage oriented frame and splits the problem in two loops: a first rotor current loop, and an outer power control loop, see details in [13], pp. 2-18 to 2-22. At this point, two main differences arises with respect to the proposed controller (33)-(34): the stability of the PQ method is based on the assumption of a fast inner loop with respect the power control loop, and the use of current sensors for the rotor side is required. Figure 7, compares the behavior of the stator currents

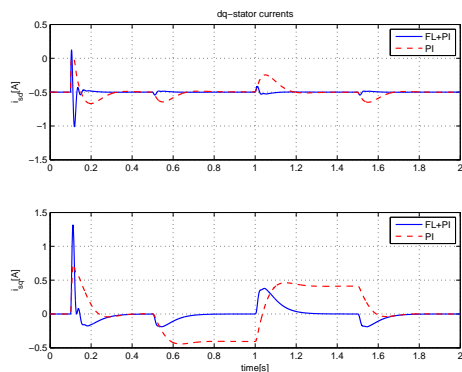


Fig. 6. Simulation results: stator currents under a change of the mechanical speed, from  $300\text{rad/s}^{-1}$  to  $295\text{rad/s}^{-1}$  at  $t = 0.1\text{s}$ , and acceleration of  $50\text{rad/s}^{-2}$  and  $-50\text{rad/s}^{-2}$ , from  $t = 0.5\text{s}$  to  $t = 1\text{s}$ , and  $t = 1\text{s}$  to  $t = 1.5\text{s}$ , respectively.

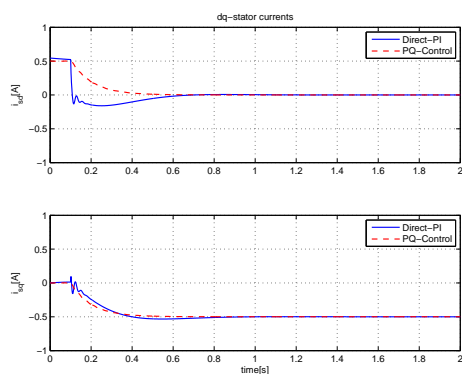


Fig. 7. Simulation results: comparison of the direct PI-controller algorithm with the stator voltage PQ control.

using the proposed algorithm versus the PQ control approach in [13], where the control gains were set to  $k_{Pp} = 0.01\text{AW}^{-1}$ ,  $k_{Ip} = 10\text{AW}^{-1}\text{s}^{-1}$  (for the PQ controller) and  $k_{Pc} = 5\text{VA}^{-1}$ ,  $k_{Ic} = 0.1\text{VA}^{-1}\text{s}^{-1}$  (for the rotor current controller). Notice that performance is similar although the proposed control law is easier to implement and less complex.

The third set of simulations test the direct PI controller. In this case, the model is implemented using the SimPowerSystems toolbox of Matlab, which contains a library with realistic implementations of some elements such as sources, electrical machines and measurement elements. For this simulation, the Blondel-Park transformations had to be coded, as well as the obtained control action,  $v_r$ , that is converted to the three-phase voltage,  $V_r$ , to be applied to the rotor side of the DFIM. Also, the effects of a real implementation are included, *i.e.* sampling, quantification and saturation. The sampling frequency is set at  $10\text{kHz}$ , all the variables are quantified as in a 16bit processor, and the rotor voltages which are saturated at  $\pm 120\text{V}$ , are delayed with one sampling time period. The control parameters are  $k_P = 5\text{VA}^{-1}$  and  $k_I = 50\text{VA}^{-1}\text{s}^{-1}$ . This simulation is performed with the same parameters used in Section VII (which consists in a small DFIM with large stator and rotor resistances), but a test using five times smaller  $R_s$  and  $R_r$  values, is also provided in order to show the performance in a large DFIM.

Figure 8 shows how the obtained dq-stator currents stabilize at the desired values under several reference changes. Not using feedback linearizing terms implies that the dq-currents are coupled and changes on  $i_{sd}$  affect in  $i_{sq}$  (and vice-versa), but the currents recover the set point. It is worth mentioning that the controller is able to operate for both signs of the  $i_{sd}$  current,

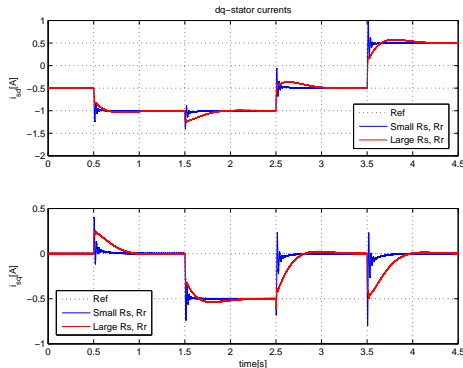


Fig. 8. Simulation results: dq-stator currents depending on the DFIM resistance values.

showing that this algorithm can be used for both generation of either electrical power or mechanical power (as in driving applications). As expected because the closed loop poles depend on the resistance values, with the same control gains, the response of a DFIM with small resistances is faster (with an overshoot) than the machine with larger ones.

## VII. EXPERIMENTAL RESULTS

The proposed controller has been tested experimentally as well. The DFIM is a 1.1kVA, 2 poles three-phase machine (DeLorenzo DL 1022K), with the following parameters:  $R_s = 4.92\Omega$ ,  $R_r = 4.42\Omega$ ,  $L_s = 725\text{mH}$ ,  $L_r = 715\text{mH}$  and  $L_{sr} = 710\text{mH}$ . The DFIM is moved by a 3kW DC motor used to provide a constant speed (at 2950rpm). Assuming a balanced grid, by using two voltage differential sensors and two Hall effect currents sensors, the three phase stator voltages and currents are measured. Position is measured as well in order to compute the dq transformation. The sample time is fixed to  $10^{-4}\text{s}$ , which corresponds to 10kHz as a maximum frequency.

The first test corresponds to a change on the d-stator current component. The DFIM starts with references at  $i_s^* = [0, 0]\text{A}$  and the d-component is set to  $i_{sd}^* = 0.5\text{A}$  (Figure 9) and changed back to  $i_{sd}^* = 0\text{A}$  (Figure 10). Figures 9 and 10 show the transient of the dq-stator currents and its references. Small oscillations of the transformed dq-currents corresponds to the effect of unbalanced phases of the actual machine. As in the simulation tests, the controller is able to control the active power by means of the d-stator current component. However, the transients of the stator currents are worst than the ones obtained in the simulations due to the interaction between the control required to keep the mechanical speed of the DC machine and the tested control algorithm. In a practical scenario with larger inertia values than the one used in the experiment, the mechanical time constant will smooth the current oscillations. Figure 11 shows the stator voltage and current for the a-phase for  $i_s^* = [0.5, 0]\text{A}$ . Note that as the reactive power in the stator side is zero ( $i_{sq} = 0$ ), the stator voltage and current are in phase.

The second test consists in to modify the reactive power of the DFIM. In this case the q-stator current component changes from  $i_s^* = [0.5, -0.5]\text{A}$  to  $i_s^* = [0.5, 0.5]\text{A}$ . Figure 12 shows the a-phase stator voltage and current at the beginning of the test where the phase of the current lags that of the voltage. The dq-stator currents behavior is shown in Figure 13. This test shows that the reactive power can be modified using the q-stator current component. Finally, the a-phase stator voltage and current in steady state are in Figure 14. In this case, the voltage lags the current. As the change in the q-component does not affect the active power, the mechanical speed is only slightly affected and the current performances are better than the previous cases.

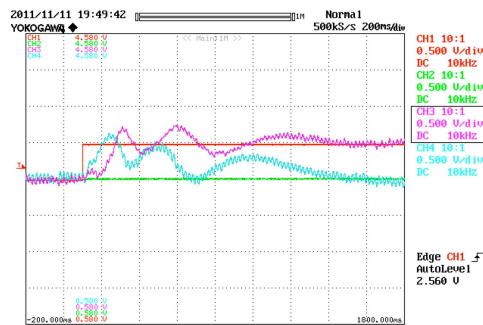


Fig. 9. Experimental results: dq-stator currents for a change reference from  $i_s^* = [0, 0]A$  to  $i_s^* = [0.5, 0]A$ . (CH1): d-stator current reference. (CH2): q-stator current reference. (CH3): d-stator current. (CH4): q-stator current.

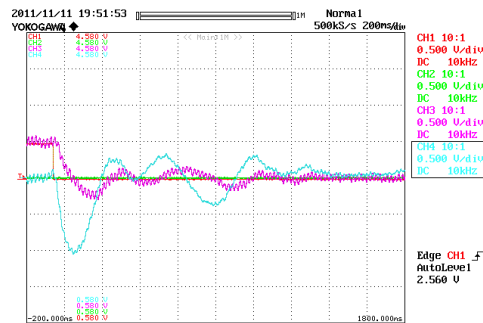


Fig. 10. Experimental results: dq-stator currents for a change reference from  $i_s^* = [0.5, 0]A$  to  $i_s^* = [0, 0]A$ . (CH1): d-stator current reference. (CH2): q-stator current reference. (CH3): d-stator current. (CH4): q-stator current.

## VIII. CONCLUSIONS

In this paper, a particularly simple controller for the DFIM was presented. It consists of a PI regulator for the stator currents and (possibly) a feedback linearizing term. As the proposed scheme is defined in the stator voltage reference frame, the active and reactive powers are directly related to the d and q stator currents, respectively, and the power regulation does not require extra loops or computations. Moreover, no stator flux estimation is required. Consequently, the algorithm is simpler than classical vector control.

In contrast with the standard decoupling controllers, the PI action is defined with a particular structure (relating the d-rotor

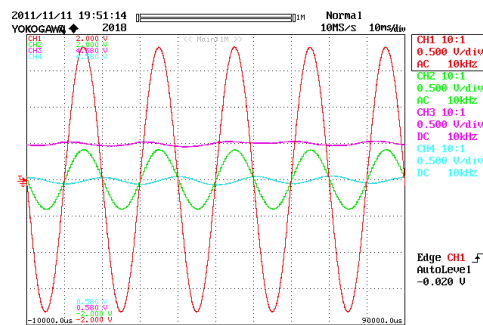


Fig. 11. Experimental results: a-phase stator voltage (CH1), a-phase stator current (CH2), d-stator current (CH3) and q-stator current (CH4), for  $i_s^* = [0.5, 0]A$ .

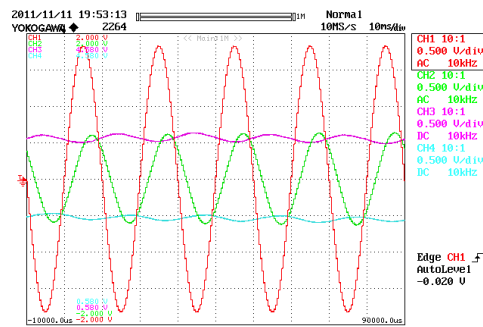


Fig. 12. Experimental results: a-phase stator voltage (CH1), a-phase stator current (CH2), d-stator current (CH3) and q-stator current (CH4), for  $i_s^* = [0.5, -0.5]$ A.

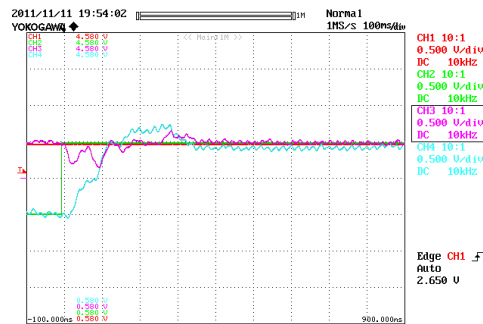


Fig. 13. Experimental results: dq-stator currents for a change reference from  $i_s^* = [0.5, 0.5]$ A to  $i_s^* = [0.5, -0.5]$ A. (CH1): d-stator current reference. (CH2): q-stator current reference. (CH3): d-stator current. (CH4): q-stator current.

voltage component with (minus) the error of the q-stator current, and the q-rotor voltage with (plus) the error of the d-stator current). Stability is analyzed with a Routh-Hurwitz test for polynomials with complex coefficients. This method provides a simple analysis tool to determine the stability regions of the control gains.

The paper presents two approaches. The first algorithm consists of a feedback linearization stage plus a PI action. This scheme is particularly attractive because the resulting system is linear and independent of the mechanical speed. The influence of an incorrect estimation of the rotor resistance is also studied. The second algorithm only uses the PI term, so that it is extremely

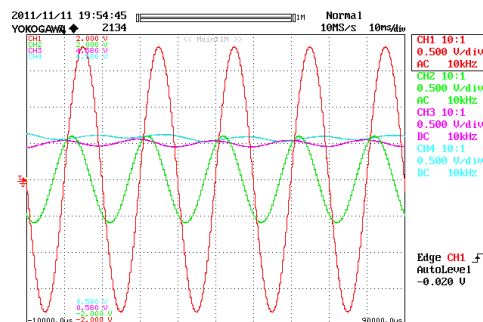


Fig. 14. Experimental results: a-phase stator voltage (CH1), a-phase stator current (CH2), d-stator current (CH3) and q-stator current (CH4), for  $i_s^* = [0.5, 0.5]$ A.

easy to implement. This current loop does not require either the knowledge of the machine parameters or measurements of the rotor currents. As opposed to the previous algorithm, the stability of the second approach is based on the assumption of constant mechanical speed. However, this restriction is widely assumed and can be accepted when the time constant of the mechanical dynamics is much higher than the electrical one. As a consequence of using a direct regulation of the stator currents (measurements of the rotor currents are not required), neither of the proposed algorithms allows to assign the closed loop poles, that are dependent on the DFIM parameters.

The proposed controllers are verified in simulations. First, a dq-model of the DFIM is used to compare both controllers, resulting in a similar behavior. Secondly, a comparison is done with respect to the common PQ control. The presented algorithm is found easy to tune and cheaper to implement because it does not requires sensors for the rotor currents. And third, the direct PI algorithm is tested in a more accurate scenario. The model used in the simulation contains some parasitic elements and non-ideal effects such as sampling, quantification and some delay, in order to emulate a real experiment. The result demonstrate good performance and validate the proposed control scheme. Finally, some experimental results using the direct PI algorithm for a real setup are provided. These experimental tests confirm the advantages predicted in the simulation stage.

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## APPENDIX

**Appendix A: Parameters of the  $\Delta_2$  and  $\Delta_3$  conditions in Subsection IV-B.** Parameters of (29) and (30) are

$$\begin{aligned} c_1 &= -\mu L_{sr}^2 & c_2 &= L_{sr}^2 \beta & c_3 &= -2\omega_s \mu L_{sr} L_r R_s \\ c_4 &= \omega_s L_{sr} L_r R_s \beta & c_5 &= R_s \tilde{R}_r (\beta^2 + \omega_s^2 \mu L_s L_r) \end{aligned}$$

and

$$\begin{aligned} d_1 &= -\omega_s \mu L_{sr}^3 R_s \tilde{R}_r & d_2 &= \omega_s L_{sr}^3 R_s (\tilde{R}_r \beta - \omega_s^2 \mu L_r) \\ d_3 &= -\omega_s^2 \mu L_{sr}^2 R_s (L_s \tilde{R}_r^2 + \omega_s^2 \mu L_r + 3L_r R_s \tilde{R}_r) & d_4 &= \omega_s^3 L_{sr}^3 L_r R_s \beta \\ d_5 &= \omega_s^2 L_{sr}^2 R_s \tilde{R}_r (\omega_s^2 \mu L_s L_r + \beta^2 + L_r R_s \beta) & d_6 &= \omega_s L_{sr} R_s \tilde{R}_r^2 (\omega_s^2 \mu L_s L_r + \beta^2) \end{aligned}$$

where  $\beta = L_r R_s + L_s \tilde{R}_r$ .

**Appendix B: Parameters of the complex polynomial  $\det \mathcal{A}(s)$  in Section V.** Parameters of (20) in Section V are  $a_0 = \mu$ ,

$$\begin{aligned} a_1 &= L_r R_s + L_s \tilde{R}_r & a_2 &= R_s R_r - \omega_s (\omega_s - \omega) \mu + k_P \omega_s L_{sr} & a_3 &= k_I \omega_s L_{sr} \\ b_1 &= \mu (2\omega_s - \omega) - k_P L_{sr} & b_2 &= (\omega_s - \omega) L_r R_s + \omega_s L_s R_r - k_I L_{sr} & b_3 &= 0 \end{aligned}$$

**Appendix C: Parameters of the  $\Delta_2$  and  $\Delta_3$  conditions in Section V.** Parameters of (29) and (30) in Section V are

$$\begin{aligned} c_1 &= -\mu L_{sr}^2 & c_2 &= L_{sr}^2 \gamma & c_3 &= \mu L_{sr} (\omega (L_s R_r - L_r R_s) - \omega_s \gamma) \\ c_4 &= \omega L_{sr} L_r R_s \gamma & c_5 &= R_s R_r \gamma^2 \end{aligned}$$

and

$$\begin{aligned} d_1 &= -\omega_s \mu L_{sr}^3 R_s R_r & d_3 &= \omega_s \mu L_{sr}^2 R_s (-2\omega_s R_r \gamma - \omega_s \omega^2 \mu L_r - \omega R_r (L_r R_s - L_s R_r)) \\ d_2 &= \omega_s L_{sr}^3 R_s (R_r \gamma - \omega_s \omega \mu L_r) & d_5 &= \omega \omega_s^2 \mu L_{sr}^2 L_r R_s (\omega L_s R_r - (\omega_s - \omega) \gamma) + \omega_s L_{sr}^2 R_s R_r \gamma (\omega_s \gamma + \omega L_r R_s) \\ d_4 &= \omega \omega_s^2 L_{sr}^3 L_r R_s \gamma & d_6 &= \omega_s L_{sr} R_s R_r (R_s R_r - (\omega_s - \omega) \omega_s \mu) (\gamma^2 + \omega^2 \mu L_s L_r) \end{aligned}$$

where  $\gamma = L_r R_s + L_s R_r$ .