Midterm Exam I

Oct. 3, 8:35-9:25am

Name: Elvis

(42 points total)

PROBLEM 1: Consider the passive network shown in Fig. 1, where we are interested in the voltage gain \( v_{out}/v_{in} \). [9 points]

![Passive Network Diagram](image)

Figure 1: Passive network for Problem 1.

(a) What is the low frequency voltage gain \( v_{out}/v_{in} \) expressed in dB? [3]

(b) Estimate the 3-dB frequency (in rad/s) for \( v_{out}/v_{in} \) using the method of open circuit time constants. [6]

\[ R_1 = 10 \, \text{k}\Omega, \quad R_2 = 10 \, \text{k}\Omega, \quad C_1 = 2 \, \mu\text{F}, \quad C_2 = 1 \, \mu\text{F}, \quad R_3 = 20 \, \text{k}\Omega \]

1. At low freq:

   \[ \frac{v_{out}}{v_{in}} \text{ entire signal appears at output, } \]

   \[ 20 \cdot \log(1) = 0 \, \text{dB} \]

2. \[ R_{cl} = R_1 = 10 \, \text{E}3 \]

   \[ T_{cl} = 2 \, \text{E} - 6 \cdot 10 \, \text{E}3 = 0.02 \]

   \[ R_{cl} = R_1 + R_2 + R_3 = 4 \, \text{E}3 \]

   \[ T_{cl} = 1 \, \text{E} - 6 \cdot 4 \, \text{E}3 = 0.04 \]

   \[ \omega_{3\text{dB}} \approx \frac{1}{T_{cl} + \frac{C_1}{C_2}} \approx \frac{1}{0.04 \cdot 10 \, \text{E}3 + 1} \approx 16.7 \, \text{rad/s} \]
PROBLEM 1 (cont'd)
PROBLEM 2: Consider the amplifier shown in Fig. 2, where \( \frac{W}{L}_1 = 10 \) and \( \frac{W}{L}_2 = 5 \). Assume that for all transistors, \( V_c = 1 \text{ V}, \mu C_{ox} = 2 \text{ mA/V}^2 \), \( V_A = 20 \text{ V}. \) Also, the power supply voltage is \( V_{DD} = 10 \text{ V} \) and the large signal component of the input voltage is \( V_{IN} = 2 \text{ V}. \) Note that \( M_2 \) is diode connected (this is not the standard common source amplifier with active load that we have analyzed in class). [1\# points]

Figure 2: Amplifier for Problem 2.

(a) What is the steady state value of the drain current of the transistors (\( I_{D1} = I_{D2} \))? You may ignore channel length modulation for both transistors in this part. [2]

(b) What is the large signal component of the output voltage (\( V_{OUT} \))? You may ignore channel length modulation for both transistors in this part. [3]

(c) What is the small signal gain of the amplifier? You may ignore channel length modulation for \( M_2 \) only in this part. [9]

\[
\begin{align*}
(I_{D1})_{SS} &= \frac{\mu C_{ox} W}{2} \left( V_{GS1} - V_T \right)^2 \\
&= \frac{2E_3 \cdot 10 \cdot (2-1)^2}{2} = 10 \text{ mA}.
\end{align*}
\]

(b) Find \( V_{GS2} \): \( 10E_3 = \frac{2E_3}{2} \cdot 5 \left( V_{GS2} - 1 \right)^2 \) \( \Rightarrow V_{GS2} = \sqrt{2} + 1 = 2.41 \)

\[ V_{OUT} = V_{D1} - V_{GS2} = 10 - 2.41 = 7.59 \text{ V} \]
PROBLEM 2 (cont'd)

(c) Draw SS. model:

\[ U_{in} \quad U_{out} \quad U_{out} \quad \frac{1}{g_{m2}} \quad \frac{1}{g_{m1}} \quad U_{in} \]

\[ \frac{U_{out}}{U_{in}} = -g_{m1} \left( g_{m1} \parallel \frac{1}{g_{m2}} \right) \]

Now, \[ g_{m1} = \sqrt{2 \mu C_L W I_0} = \sqrt{2 \cdot 2E-3 \cdot 10 \cdot 10E-3} = 20E-3 \]

\[ g_{m2} = \sqrt{2 \cdot 2E-3 \cdot 5 \cdot 10E-3} = 10 \sqrt{2} = 14.1E-3 \]

\[ R_0 = \frac{V_A}{I_0} = \frac{20}{10E-3} = 2E3 \]

\[ R_0 \parallel \frac{1}{g_{m2}} = \left( \frac{1}{2E3} + 14.1E-3 \right)^{-1} \approx 68.5 \, \Omega \]

\[ \frac{U_{out}}{U_{in}} = -20 \cdot 10E-3 \cdot 68.5 = -1.37 \]
PROBLEM 3: Consider an amplifier with a transfer function $A(s)$ that has a zero at $s = -10^6$ rad/s, poles at $s = -10^4$ rad/s and $s = -10^7$ rad/s, and a low frequency gain of $A(0) = 60$ dB. [9 points]

(a) What is the transfer function $A(s)$? [3]

(b) Sketch the magnitude and phase response bode plots for this amplifier on the plots provided in Fig. 3. [6]

\[ A(s) = \frac{i\infty}{(1 + s/10^6)} \frac{(1 + s/10^7)}{(1 + s/10^4)} \]

Figure 3: Bode plots for Problem 3.
PROBLEM 4: Consider the differential pair shown in Fig. 4. The pair is biased with \( I_{REF} = 10 \text{ mA} \), and are sized so that \( g_{m1} = g_{m2} = 20 \text{ mA/V}^2 \). You may ignore all channel length modulation effects in this problem. [8 points]

![Differential Pair Diagram]

Figure 4: Differential pair for Problem 4.

(a) What is the differential offset voltage at the output? [4]
(b) What is the input-referred differential offset voltage? Use the nominal resistor value \( R_D = 1 \text{ k}\Omega \) in your gain calculation. [4]

\[ V_{out^+} = V_{DD} - \frac{I_{REF} \cdot R_P}{2} = V_{DD} - 5 \cdot 10^{-3} \cdot 1 \cdot 10^3 = V_{DD} - 5.5 \]

\[ V_{out^-} = V_{DD} - 5 \cdot 10^{-3} \cdot 1.0 \cdot 10^3 = V_{DD} - 5.0 \]

\[ V_{offset_{d,DP}} = 0.5 \text{ V} \]

b) Gain of differential pair = \( g_{m1} \cdot R_P = 20 \cdot 10^{-3} \cdot 1 \cdot 10^3 = 20 \).

Input referred offset voltage:
\[ V_{offset_{1,10}} = \frac{0.5 \text{ V}}{20} = 25 \text{ mV} \]
PROBLEM 5: Consider the four famous scientists shown in Fig. 5. Who was the only one NOT to win the Nobel Prize in Physics? [2 points]

(a) Schrödinger  
(b) Einstein  
(c)  
(d) Planck

(c) Mr. Borat Sagdiyev

Figure 5: Renowned scientists.