Example: We want to design a dual-slope ADC to digitize an analog signal with a 5 V range. Assume power supplies are ±5 V, clock 10 MHz, opamp has input leakage current of 10 μA, and can source a max. current of 5 mA at output.

Requirements:
1. Quantization error \( \leq 10 \text{ mV} \).
2. Error due to leakage should be \( \leq 1 \text{ LSB} \).

3. How many bits do we need?

   - Max. quantization error is \( \frac{1}{2} \text{ LSB} \). \( \Rightarrow 1 \text{ LSB} = \frac{5 \text{ V}}{2^{N-1}} \)

   \[
   \frac{1}{2} \left( \frac{5 \text{ V}}{2^{N-1}} \right) \leq 10 \text{ mV} \\
   2^{N-1} \geq 250
   \]

   - Choose \( N = 8 \), \( 2^8 - 1 = 255 > 250 \)

   \( \therefore \) We will design an 8-bit ADC.

4. What is the maximum frequency signal we can digitize without aliasing?

   - Clock is 10 MHz, \( \text{Tick} = \frac{1}{10 \text{E}6} = 0.1 \mu \text{s} \)

   - Output \( n \) of counter must vary from 0 to \( \text{NREF} \) with 256 levels,
     \( \therefore \text{NREF} = 255 \)

   - Worst case conversion time is when \( n = \text{NREF} \), so counter needs \( 2(\text{NREF}+1) \) clock cycles to complete.

     \( \Rightarrow \) Total conversion time: \( T_c = 2(\text{NREF}+1) \times \text{Tick} \)

   - Max. sample rate: \( \frac{1}{T_c} = \frac{1}{2(\text{NREF}+1) \times \text{Tick}} = \frac{\text{Tick}}{2(\text{NREF}+1)} = \frac{10 \text{Mhz}}{2(256+1)} = 19.5 \text{ kHz} \)

   \( \Rightarrow \) To avoid aliasing, we need to sample at \( \geq 2 \times \text{the signal bandwidth} \)

   - Max. freq. signal we can digitize is \( \frac{19.5 \text{ kHz}}{2} = 9.8 \text{ kHz} \).

5. Choose R4C to satisfy the requirements.
Need to choose a max $V_{\text{PEAK}}$ that we will charge up to

- Choose $V_{\text{PEAK}}$ as positive supply rail to reduce susceptibility to leakage (more on this later).

$V_{\text{PEAK, max}} = 5 \text{ V}$

- We know that after $T_1$, $V_{\text{PEAK}} = \frac{V_a}{RC}$

$V_{\text{PEAK, max}}$ should be reached for max value of $V_a$ input.

- $5 \text{ V} = \frac{5 \text{ V}}{R_C}$ (Assuming $V_{\text{REF}} = 5 \text{ V}$)

$RC = \frac{V_{\text{REF}}}{I_{\text{ref}}} = \frac{255}{1.1 \times 10^{-7}} = 25.5 \times 10^6 = RC$

- This gives us a relation between $R$ & $C$, now let’s choose values.

- To do this we must consider other requirements.

- First consider max opamp current:
  - Consider behavior during discharge

$V_{\text{REF}} \rightarrow \frac{I}{C} \rightarrow \text{GND}$

- Current $I = \frac{V_{\text{REF}}}{R}$

- This current must be sunk by opamp, which can sink a max of $5 \text{ mA}$

$\therefore V_{\text{REF}} \leq 5 \text{ mA} \Rightarrow R \geq \frac{5 \text{ V}}{5 \text{ mA}} = 1 \text{ k}\Omega$

- So, $R$ has a lower bound of $1 \text{ k}\Omega$

- What about an upper bound?

- Now consider leakage current:
  - For simplicity, we will only consider leakage that occurs during the $T_1$ phase (otherwise equations get messy).
In the ideal case, during charging we have \( V(t) = \frac{(VA) + 1}{RC} \cdot \frac{1}{C} = \frac{VA \cdot t}{RC} \).

- With leakage current, IL adds to IR to increase the total current Ic charging the capacitor.

Now, \( V(t) = \left( \frac{VA + IL}{R} \right) \cdot \frac{t}{C} \)

This increases the slope of the Vi phase, causing \( V_{peak} \) to reach a higher value than it should.

\[ \Delta V_{peak} \] (error) - now, this takes longer to discharge, resulting in an error \( \Delta T_2 \).

- We require that \( \Delta T_1 \leq T_{clk} \) (1 LSB).

- First find \( \Delta V_{peak} \) (constant): \( \Delta V_{peak} = \frac{IL \cdot T_1}{C} = \frac{IL \cdot \text{REF} \cdot T_{clk}}{C} \)

- Now find \( \Delta T_2 \): - requesting leakage during second phase, we have:
  \[ V_I = V_{peak} - \frac{V_{\text{ref}} \cdot t}{RC} \]

- Set \( V_I = 0 \), set \( V_{peak} = \Delta V_{peak} \), solve for \( t = \Delta T_2 \).

\[ 0 = \frac{IL \cdot \text{REF} \cdot T_{clk} - V_{\text{ref}} \cdot \Delta T_2}{RC} \]

\[ \Delta T_2 = \frac{IL \cdot \text{REF} \cdot T_{clk} \cdot R}{V_{\text{ref}}} \]

Now, set \( \Delta T_2 \leq T_{clk} \) and solve for \( R \): \( 10^{-0.5 \cdot 2.55 \cdot \text{REF} \cdot R} \leq T_{clk} \)

\[ R \leq 1.96 \, k\Omega \]

- So, need to choose \( 1.0 \, k\Omega \leq R \leq 1.96 \, k\Omega \). \( \Rightarrow \) Choose \( R = 1.5 \, k\Omega \)

- Now, \( RC = 25.5 \times 10^{-6} \Rightarrow C = 17 \, nF \)

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