Quantization takes the continuous value that was sampled in the previous step and represents it by the closest discrete level.

Eq: 3 bits → 8 levels

\[ \text{cont. value} \rightarrow \text{discrete value} \]

If we represent a 0-5 V continuous value signal with a 3-bit digital word, the step size between levels is \( \frac{5V}{8} = 0.71V \) \( \frac{5V}{(2^3-1)} \)

This quantity is referred to as the Least Significant Bit, and determines how much quantization error we will have.

The higher the number of bits in the ADC, the smaller the quantization error.

The above process is carried out by ADCs to move from the analog domain to the digital domain.

The reverse procedure is carried out by DACs to go from the digital to the analog domain.

- Represent a signal by discrete levels at "sampling" intervals.
- Additional filtering must be performed to reconstruct the signal.

Discontinuities in DAC output are at high frequency compared to signal being reconstructed.

Low-pass filtering restores original signal.

Observe that information lost to quantization error cannot be recovered, as the reconstructed signal will differ from the original due to this.

Functional blocks

\[ \text{A/D} \rightarrow \text{digital word} \rightarrow \text{D/A} \rightarrow \text{signal} \]
DAC Circuits (Topic of Lab 4)

Binary-Weighted Resistor

- Switches are controlled by a digital input word: \( b_1, b_2, b_3 \ldots b_N \)
  - \( b_i \) determines the position of \( S_i \): \( b_i = 0 \), \( S_i \) is in left position (ground)
  - \( b_i = 1 \), \( S_i \) is in right position (Vref input)
- Current flowing from Vref remaining constant.

\[ I_0 = \frac{V_{\text{ref}}}{R} \left(b_1 + \frac{V_{\text{ref}}}{2R} \cdot 2^{1} \cdot 2^{N-1} \cdot R + \ldots + \frac{V_{\text{ref}}}{2^{N-1}} \cdot R \right) = \frac{2 \cdot V_{\text{ref}}}{R} \cdot D \]

Define \( D \), ranges from 0 to 1 - under control of digital input word.

\[ V_0 = -\frac{I_0 \cdot R}{2} = -\frac{2 \cdot V_{\text{ref}} \cdot D \cdot R}{2} = -V_{\text{ref}} \cdot D \]

- Output range from 0 to \(-V_{\text{ref}}\) under control of digital input word.

Notes:
1. Accuracy depends on:
   - Accuracy of \( V_{\text{ref}} \)
   - Accuracy of \( R \)
   - Perfect switches (zero on-resistance)
2. Accuracy suffers for large \( N \).

Example: Consider a 5-bit DAC with resistors with \( 1\% \) accuracy.
   - Ideally: currents are \( \frac{b_1 + b_2 + \ldots + b_5}{2^5} = \frac{2}{16R} \left(0.5b_1 + 0.25b_2 + \ldots + 0.03125b_5\right) \)
   - With mismatches: currents are \( \frac{b_1 + b_2 + \ldots + b_5}{0.98765\ldots16R} \)

\[ = \frac{2}{R} \left(\frac{b_1 + b_2 + \ldots + b_5}{3.5} \right) \]

\[ = D \]

\[ \Rightarrow D = 0.556b_1 + 0.25b_2 + \ldots + 0.028b_5 \]

- Error in MSB is twice the LSB, run the risk of non-monotonic output
- Effectively we only have a 4-bit DAC.
R-2R Ladder based DAC

\[ I_{n+1} = 2I_n \]

<table>
<thead>
<tr>
<th>Resistance looking into A is ( R_A = 2R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Looking into B, ( R_B = R + R_A = 3R )</td>
</tr>
<tr>
<td>Looking into C, ( R_C = 2R )</td>
</tr>
</tbody>
</table>

- By proof by induction (?), resistance to the right of each vertex is \( 2R \)
- Now consider currents, current in each resistor of \( N^{th} \) section will be equal, say \( I_n \)
- Also, current in the resistor feeding the \( N^{th} \) section will be \( 2I_n \)
- Similarly, current in each resistor of the \((N-1)^{th}\) section will be equal, \( \frac{V_{\text{out}}}{2} \) \( I_{n-1} \)
- Let this be the resistor feeding the \( N^{th} \) section, so \( I_{n-1} = 2I_n \)
- We can again follow the chain to show \( I_1 = 2I_2 = 4I_3 \), etc.

We can use this information to build a DAC:

We again use the control word \( b_1 b_2 b_3 \ldots b_N \) to represent the position of the switches:

\[ I_0 = b_1 I_1 + b_2 I_2 + \ldots + b_N I_N \]

\[ = b_1 I_1 + b_2 I_2 + \ldots + b_N \frac{I_1}{2^{N-1}} \]

\[ = \frac{V_{\text{ref}}}{R} \left( \frac{b_1}{2^1} + \frac{b_2}{2^2} + \ldots + \frac{b_N}{2^N} \right) \]

\[ \therefore V_0 = -I_0 R = -\left( \frac{V_{\text{ref}}}{R} \right) D \cdot 2^R = -V_{\text{ref}} D \] (same as for binary-weighted)