\[ I_{D1} = \frac{V_{DD} - V_{GS}}{R} \]

- Combine (1) and (2) to find \( V_{GS} \) and \( I_{D1} \).

- Now, \( V_{GS} \) for \( M2 \) = \( V_{GS} \) for \( M1 \), so as long as \( M2 \) has a \( V_{GS} \) high enough to keep it in saturation,

\[ I_{D2} = \frac{M_2 \cdot L \cdot W \cdot (V_{GS} - V_{T})^2}{2} \]

\( (W/L) \) is very useful for replicating a bias current at different points in a circuit.

- Can plot output current vs. output voltage: \( (\text{just } I_{D2} \text{-} V_{GS} \text{ relationship of } M2) \).

\[ \text{finite slope due to finite output impedance (} = R_o \text{ of } M2 \text{).} \]

- This has assumed we need to "sink" a current of \( I_{S}\text{eff} \), if we need to "source" that same current, we can use PMOS devices:

- Same circuit can be used with NFETs, although \( I_{S}\text{eff} = I_o \text{ relationship is not as simple due to finite base current.} \)

MOS Differential Pair

- Big reason to use differential amplifier is reduced sensitivity to noise & interference.

- Diff. amps are well-suited for integrated circuits due to good matching between devices.
Operation 1: Common Mode Input Voltage

- Let $V_{o1} = V_{o2} = V_{cm}$

  then, $V_{os1} = V_{os2} = V_{cm} - V_s$

- Current will split equally between $M_1$ and $M_2$, $V_{o1} = V_{o2}$
  - prove by assuming this is not true.

- If $V_{cm}$ changes, current will still split equally, no change in differential output voltage $(V_{o1} - V_{o2})$.

Definition: input common mode range: range of $V_{cm}$ over which the pair can operate.

- Limited at high end by $M_1$, $M_2$ entering the triode region.
- Limited at the low end by need for adequate voltage across arm source (not a problem for ideal source, but could be for actual mirror).

Operation 2: Differential Mode Input Voltage

- Let $V_{o1} = +V_{id}$, $V_{o2} = -V_{id}$.

  - can see that current flow will no longer split equally, more current will flow in $M_1$ (reducing $V_{o1}$) and less in $M_2$ (increasing $V_{o2}$).

- What value of $V_{id}$ will cause all the current to flow in one side?

  - at this point, must satisfy eqn: $I = \frac{1}{2} K_n V_d^2 (V_{os1} - V_t)^2$.

  \[(1) \Rightarrow V_{os1} = V_t + \sqrt{2 \frac{I}{K_n} (V_{id})}\]

  - Now, if no current flows in $M_2$, $V_{os2} = V_t$ (in cutoff).

  - $V_{os2} = V_{o2} - V_s = V_d - \frac{V_{id}}{2} - V_s = V_t$

    \[\therefore V_s = \frac{V_{id} - V_t + V_d}{2}\]
Now, \( V_{0s1} = V_{01} - V_S = \frac{V_{id}}{2} - V_S = \frac{V_{id}}{2} - (\frac{V_{id}}{2} - V_t) = V_{id} + V_t. \)

\[ \text{Solve back into (1): } V_{id} + V_t = V_t + \sqrt{\frac{2-V_t}{k_n'(W/L)}} \]

\[ \Rightarrow V_{id} = \sqrt{\frac{2-V_t}{k_n'(W/L)}} = \sqrt{V_t - V_{ov}} \text{ (overdrive voltage with current of } \frac{1}{2} \text{)} \]

- If \( V_{id} \) increases beyond this point, we will see no additional change in output voltage.
- Need to operate in range \(-\sqrt{V_t - V_{ov}} \leq V_{id} \leq \sqrt{V_t - V_{ov}} \).

Large Signal Operation

- Derive exact relationship between \( V_{id} \) and \( i_{01}, i_{02} \).
- Gloss over some details, see pages 693–694 in text for full version.

\[
\begin{align*}
V_{0b} + \frac{V_{id}}{2} & \quad \text{(1)} \\
\text{(Vb)} & \quad \text{(2)} \\
\end{align*}
\]

\[ i_{01} = \frac{1}{2} k_n' W \left( V_{0s1} - V_b \right)^2 \]

\[ i_{02} = \frac{1}{2} k_n' W \left( V_{0s2} - V_b \right)^2 \]

Taking square roots:

\[ \sqrt{i_{01}} = \frac{1}{2} k_n' W \left( V_{0s1} - V_b \right) \quad (1) \]

\[ \sqrt{i_{02}} = \frac{1}{2} k_n' W \left( V_{0s2} - V_b \right) \quad (2) \]

Now, \( V_{0s1} - V_{0s2} = V_b + \frac{V_{id}}{2} - V_S = (V_b - \frac{V_{id}}{2} - V_S) = V_{id}. \quad (3) \)

Subtract (2) from (1) and subtract (3) to get:

\[ \sqrt{i_{01}} - \sqrt{i_{02}} = \frac{1}{2} k_n' W \frac{V_{id}}{L} V_{id}. \quad (4) \]

Also, the current source imposes the constraint, \( i_{01} + i_{02} = I \) \( \text{(5)} \)

- We have 2 equations with 2 unknowns!
- Solve for \( i_{01}, i_{02} \) (details in book).

We find that:

\[ i_{01} = \frac{1}{2} + \frac{1}{2} \left( \frac{I}{V_{ov}} \right) \left( \frac{V_{id}}{2} \right) \sqrt{1 - \left( \frac{V_{id}/2}{V_{ov}} \right)^2} \]

\[ i_{02} = \frac{1}{2} - \frac{1}{2} \left( \frac{I}{V_{ov}} \right) \left( \frac{V_{id}}{2} \right) \sqrt{1 - \left( \frac{V_{id}/2}{V_{ov}} \right)^2} \]

where \( V_{ov} = \frac{1}{\sqrt{k_n'(W/L)}} \), as before.