Next: Common Mode Gain $\frac{A_{cm}}{A_{cm}}$

\[ M_1, M_2 \text{ are CS transistor with a source degeneration of } 2R_{ps} \]

\[ \therefore G_m = \frac{G_m}{1 + G_m R_{deg}} \approx \frac{1}{R_{deg}} \quad \text{if} \quad G_m R_{deg} \gg 1 \]

\[ \therefore \quad V_o = \frac{i_1}{2 R_{ps}} \quad \therefore \quad i_1 = i_2 = \frac{V_o}{2 R_{ps}} \]

- To resistance looking into $M_2$
  \[ M_2 = \frac{1}{R_{ps} \| G_m} \]

- To resistance looking into $M_1$
  \[ M_1 = R_0 + \frac{(1 + G_m R_{ps}) R_{deg}}{R_0 + (1 + G_m R_{ps}) 2R_{ps}} \]

\[ \therefore \quad R_{ps} \quad \text{is much greater than} \quad R_0. \]

- $i_2$ sees \( (R_0 \| \frac{1}{G_{m}}) \) parallel with \( R_0 + (1 + G_m R_{ps}) 2R_{ps} \)

\[ \therefore \quad V_{gs3} = -i_2 \left( R_0 \| \frac{1}{G_{m}} \right) \]

\[ \therefore \quad i_2 = G_{m} \cdot V_{gs3} = G_{m} \cdot i_2 \left( R_0 \| \frac{1}{G_{m}} \right) \]

\[ \therefore \quad V_{o} = (i_2 - i_2) \cdot R_{ps} \| (R_0 + (1 + G_m R_{ps}) 2R_{ps}) \]

\[ \approx (i_2 - i_2) \cdot R_{ps} \]

\[ \approx \left[ (i_2 \cdot G_{m} \cdot \left( R_0 \| \frac{1}{G_{m}} \right) - i_2 \right] \cdot R_{ps} \]

- After some manipulation:

\[ \frac{A_{cm}}{A_{cm}} = \frac{V_o}{2 V_{ps}} = -\frac{1}{2 R_{ps}} \left( \frac{R_0}{1 + G_m R_{ps}} \right) \approx \frac{1}{2 R_{ps} G_{m}} = A_{cm} \]

\[ \therefore \quad CMRR = \frac{A_{cm}}{A_{cm}} = G_m \left( R_0 \| R_{ps} \right) \cdot 2 R_{ps} G_{m} \]

\[ \text{CMRR} \approx (G_m R_0) \left( G_m \cdot R_{ps} \right) \]

\[ \therefore \quad \text{for the resistive load taken single-ended, we had a CMRR of } G_m R_0 \]

\[ \therefore \quad \text{we have multiplied that by } G_m R_0 \quad \text{(big improvement!)} \]

\[ \text{Multi-Stage Opamp} \]

\[ \therefore \quad \text{distorted pair functions as the input to an opamp.} \]

- Gain from this stage is on the order of 10s
- For an opamp, we need a $g_m \rightarrow \infty$
- Need to add more stages to get additional gain.
Notes:
- pMOS diff. pair for input stage
- nMOS common source amplifier with active load for second stage
- Cc is a compensation capacitor (more on this later).

DC Biasing:
- M8 is biased with \( I_{ref} \), this is mirrored to M5, M7 (with ratioing of W/L ratios)
- M3 and M4 each have a current of \( \frac{I_{ref}}{2} \) (assuming \( \frac{W}{L} ) = 8 \))

\[ V_d = \frac{V_{gs}}{3} \]

- M6 is biased to have \( I_{ref} \left( \frac{W}{L} \right)_6 = \frac{I_{ref} \left( \frac{W}{L} \right)_4}{2} \) (biasing is automatic)

- to avoid an offset voltage, we need \( I_{d7} = I_{d6} \) when the inputs are equal.

\[ \text{We need} \quad I_{ref} \left( \frac{W}{L} \right)_7 = \frac{I_{ref} \left( \frac{W}{L} \right)_4}{2} \left( \frac{W}{L} \right)_4 \]

\[ \frac{\left( \frac{W}{L} \right)_6}{\left( \frac{W}{L} \right)_4} = 2 \cdot \frac{\left( \frac{W}{L} \right)_7}{\left( \frac{W}{L} \right)_5} \]

- Otherwise we will have systematic offset.

- This biasing setup eliminates the need for blocking capacitors.

Voltage Gain:
- Break circuit into 2 stages, calculate gains separately.

\[ \text{Gain of diff. input stage:} \quad A_1 = -g_{m1} (R_{o2} || R_{o4}) \]

\[ \text{Gain of common source output stage:} \quad A_2 = -g_{m6} (R_{o6} || R_{o7}) \]

Total gain: \( A = A_1 \cdot A_2 \) (usually around 1000)
Frequency Response: Before we talk about the freq. response of this circuit, let's review general freq. response concepts.

1. Circuit is L/C, low-pass characteristic.
   - Gain fell off at low freq. due to blocking capacitors.
   - Gain fell off at high freq. due to interode MOSTET caps (or transistors).
2. Most IC circuit implementations don't use blocking caps, and so the gain does not fall off at dc.

General expression for gain: \( A(s) = A_{dc} \cdot F(s) \)

- \( F(s) = \frac{(1 + s/\omega_{2n})(1 + s/\omega_{p1})(1 + s/\omega_{p2})}{(1 + s/\omega_{Zn})(1 + s/\omega_{p1})(1 + s/\omega_{p2})} \)
- \( \omega_{Zn} \) represents the \( n \)th zero frequency.
- \( \omega_{p1} \) represents the \( n \)th pole frequency.
- As \( s \to 0 \), \( F(s) \to 1 \), \( A(s) \to A_{dc} \)
- Remember that \( s = j\omega \)

Zeros: What does a zero do?

\( H(s) = \frac{1 + s/\omega_{Zn}}{1 + j\omega/\omega_{Zn}} \)

- For \( \omega \ll \omega_{Zn} \), \( (1 + j\omega/\omega_{Zn}) \approx 1 \), zero has no effect.
  - \( \text{magnitude} = 1 \), \( \text{phase} = 0^\circ \)
- For \( \omega = \omega_{Zn} \), \( (1 + j\omega/\omega_{Zn}) = (1 + j) \)
  - \( \text{magnitude} = \sqrt{1^2 + 1^2} = \sqrt{2} \)
  - \( \text{phase} = 45^\circ \)
- For \( \omega >> \omega_{Zn} \), \( (1 + j\omega/\omega_{Zn}) \approx j\frac{\omega}{\omega_{Zn}} \)
  - \( \text{magnitude} = \frac{\omega}{\omega_{Zn}} \Rightarrow 20 \cdot \log\left(\frac{\omega}{\omega_{Zn}}\right) \) increases by 20 dB/decade.
  - \( \text{phase} = 90^\circ \)
Draw Bode plot: $20 \log |H(w)|$

- A pole also has negative zeros (same except for phase effect).

Poles: What does a pole do?

1. For $w \ll w_p$, $\frac{1}{1 + j \frac{w}{w_p}} \approx 1$, pole has no effect.
   - Magnitude $= 1 \text{ (0 dB)}$, phase $= 0^\circ$

2. For $w = w_p$, $\frac{1}{1 + j \frac{w}{w_p}} = \frac{1}{1 + j}$
   - Magnitude $= \frac{1}{\sqrt{2}} = -3 \text{ dB}$
   - Phase $= -45^\circ$

3. For $w \gg w_p$, $\frac{1}{1 + j \frac{w}{w_p}} \approx \frac{1}{-j \frac{w}{w_p}}$
   - Magnitude $= \frac{1}{\frac{w}{w_p}} \Rightarrow -20 \log \left( \frac{w}{w_p} \right)$, decreases by $20 \text{ dB/decade}$
   - Phase $= -90^\circ$

$20 \log |H(w)|$

- As long as poles are adequately spaced, this makes it easy to sketch Bode plots.