3-dB frequency — often we want to know the frequency at which the gain has dropped by 3 dB.

- If one pole is at least a factor of 4 below the others, we can make a dominant pole approximation.

\[ F(s) = \left(1 + s/w_1\right) \left(1 + s/w_2\right) \left(1 + s/w_3\right) \]

- If this is not the case, we can either determine \( w_{3\text{dB}} \) graphically from a plot, or use the following formula.

**Derivation:**

Let \( F(s) = \left(1 + s/w_1\right) \left(1 + s/w_2\right) \left(1 + s/w_3\right) \)

\[ |F(s)|^2 = \frac{\left(1 + w_1^2, w_2^2\right)\left(1 + w_1^2, w_3^2\right)}{\left(1 + w_1^2, w_2^2\right)\left(1 + w_1^2, w_3^2\right)} = \frac{1}{2} \]

\[ w_{3\text{dB}} = \sqrt{\frac{1}{w_1^2, w_2^2}} - \sqrt{\frac{1}{w_1^2, w_3^2}} \]

- This is a useful approximation if we have an expression for \( F(s) \) and know all of the poles and zeros.

- What if we don't know \( w_1, 2, 3, \ldots \) and \( w_{2, 3, 4, \ldots} \)?

**Method of Open-Circuit Time Constants**

- An approximation to use when we have S.S. model, but solving for \( F(s) \) would be tedious.

**Derivation:**

- Can express \( F(s) \) as

\[ F(s) = \frac{1 + a_1 s^1 + a_2 s^2 + \cdots}{1 + b_1 s + b_2 s^2 + \cdots} \]

where \( b_1 = \frac{1}{w_1} + \frac{1}{w_2} + \cdots + \frac{1}{w_n} \)

- It can be shown (but not by us) that \( b_1 \) can be obtained as the sum of the open-circuit time constants.

- Each capacitor in the circuit contributes an open-circuit time constant.

- To find it:
  1. open circuit all other capacitances (except \( C_i \))
  2. ground input
  3. solve for the resistance seen at terminals of capacitor in question (call this \( R_{in} \))
Then, \( b_1 = \frac{n}{\sum_{i=1}^{n} C_i R_i} \),
C: open circuit time constant

- Then, we know \( b_1 \), and since \( b_1 = \frac{1}{\omega_p} + \frac{1}{\omega_{p1}} + \ldots \), it cannot
  pole exist, then \( \omega_{p1} = \omega_p = \frac{1}{b_1} \)

So, \( \omega_{p1} = \frac{1}{2\pi C_i R_i} \)

- It turns out this approximation is good even if a dominant
  pole does not exist.

Example: Use OCTC to find \( \omega_{p1} \) for common-source amplifier:

1. Draw circuit with S.S. model:

   \[
   \text{OCTC} \quad \left\{ \begin{array}{c}
   R_{\text{gs}} = \text{OCTC} \\
   R_{\text{gs}} = R_{\text{sig}} \\
   C_{\text{gs}} = C_{\text{gs}}, R_{\text{gs}} = C_{\text{gs}} R_{\text{sig}}
   \end{array} \right.
   \]

2. Find OCTC due to \( C_{\text{gs}} \):

3. Find OCTC due to \( C_{\text{gd}} \):

   - Insert test current source \( I_x \), find \( V_x \):

   \[
   V_{\text{gs}} = -I_x R_{\text{sig}}
   \]

At node D:

\[
I_x = g_{m} U_{\text{gs}} + \left( \frac{U_{\text{gs}} + V_x}{R_{L'}} \right)
\]

\[
\Rightarrow \quad R_{L'} I_x = -g_{m} I_x R_{\text{sig}} R_{L'} + I_x R_{\text{gs}} + V_x
\]

\[
\therefore R_{\text{gd}} = \frac{V_x}{I_x} = R_{L'} + R_{\text{gs}} + g_{m} R_{L'} R_{\text{gs}}
\]

\[
\therefore T_{\text{gd}} = C_{\text{gd}} R_{\text{gd}} = C_{\text{gd}} \left( R_{L'} + R_{\text{gs}} + g_{m} R_{L'} R_{\text{gs}} \right)
\]
Let's use some typical values: \( R_{S_y} = 50 \, \Omega \), \( R_L = 4 \, k\Omega \), \( C_s = 1 \, \mu F \), \( C_d = 0.2 \, \mu F \), \( \tau_m = 4 \, m\Omega / V \).

Then, \( T_{S_y} = 5 \times 10^{-11} \), \( T_{gd} = 9.7 \times 10^{-10} \)

Thus, \( \omega_H \approx \frac{1}{T_{S_y} + T_{gd}} = 9.8 \times 10^8 \) rad/s.

- How does this compare to the weighted solution?

- After some tedious KCL: \( \frac{V_{out}}{V_{in}} = \frac{R_L}{R_{S_y}} \frac{R_{S_y}(S C_d - \frac{1}{S})}{\frac{1}{S} + R_{S_y} \left[ \frac{1}{S} + \frac{1}{S} (C_d + C_s) \right] \left( 1 + S C_d R_L^2 \right) - S C_d R_L^2 (S C_d - \frac{1}{S})} \)

- Could factor this to find poles (what a nightmare!)

- Instead solve numerically using Matlab to find: \( \omega_H = 7.3 \times 10^8 \) rad/s.

- So, within about 30% of the correct answer with this approximation (but much easier to find).

- In "real life" we would use a simulator to find \( \omega_H \) (more accurate)

- So what is the value of OCTC analysis?

- It gives us design intuition

- It reveals what the dominant factors are in determining \( \omega_H \).

- We can use this intuition to alter our design for better performance.

- How the effect of \( C_d \) dominates.

- To extend B.W., we must reduce \( T_{gd} \) by altering our design.

- Note that in this case, \( C_d = \frac{C_s}{3} \) but it was the dominant effect in determining the 3dB freq.

- This is due to the Miller Effect: gain across terminals of \( C_d \) presents an effective capacitance that is larger than \( C_d \) by a factor \( \frac{1}{\omega^2} \).

\[ \text{ECE 3110: Lecture #13} \]

- Miller's Theorem:

\[ \begin{align*}
V_1 & \rightarrow I \rightarrow 2 \rightarrow V_2 = k V_1 \\
V_1 & \rightarrow 2 \rightarrow I_2 \rightarrow V_2 = k V_1
\end{align*} \]

- Replace \( \frac{1}{S} \) with equivalent capacitances "seen" by each side of the circuit.