Now let's revisit stability and take a more detailed about compensation.

- For stability, we want to apply negative feedback.
- Positive feedback implies uncontrolled growth and instability.
- Signal fed back is \(-A(s)\beta(s)\) \(V_o\).
- If quantity \(A(s)\beta(s)\) becomes negative, we now have positive feedback → instability.

- If \(A(s)\beta(s)\) is real and \(|A(s)\beta(s)| = 1\), the signal will propagate around the loop, we can remove \(V_o\) and the signal will remain at the output unless we are trying to make an oscillator, this is bad.

- So, the oscillations conspire with the condition \(A(s)\beta(s) = -1\).
- If \(A(s)\beta(s) < -1\) it will still oscillate, since non-linearities will limit the \(|A(s)\beta(s)|\) to unity for some output (e.g. 0 dB).

- So, to determine the stability of a feedback systems, we must examine \(A(s)\beta(s)\).
- We can quantify the stability of the system with the phase margin.

Plot \(\arg\{\beta(s)\}\) vs. \(\log|\beta(s)|\) of \(A(s)\beta(s)\). Check phase when \(|A(s)\beta(s)| = 1\), amount above 180° is the phase margin.

Similarly, we can also define the gain margin.

- Alternatively, since \(20 \log(A\beta) = 20 \log(A) + 20 \log(\beta) = 20 \log(A) - 20 \log(\beta)\)
- Assuming \(\beta\) is not freq. dependent, can also plot just \(20 \log(A)\), the draw in a line for \(20 \log(\beta)\), and phase margin will be that intersection of these lines.
Frequency Compensation

- To give an unstable (0° or negative phase margin) feedback system, we use freq. comp. to make it more stable (increase its phase margin).

- Freq. comp. involves either introducing new poles at low freq. or moving existing poles to lower freq.
  - This causes the gain to reach 0 dB at a lower freq., where the phase is farther from -180°.

Example:

- Initially system is unstable.
- Introduce a new pole so that the old first pole ($p_1$) hits at the gain 20 log |A(jω)| curve.
  - This will yield a phase margin of 45° (dashed line).
- System is now stable, but with a reduced gain-bandwidth product (worse performance).
- We can lessen the impact on the GBW product by moving the existing low freq. pole ($p_1$) to a lower frequency ($p_1'$) instead of introducing an additional pole ($p_c$).
  - Illustrated by dotted line in the figure.
  - Still have the phase margin, but with a higher GBW product.

- In Lab 3, we used both types of compensation:
  - New pole introduced through Miller Effect
  - Move existing pole by bleeding capacitor larger.
Let's illustrate compensation with an example, similar to lab 3.

Consider the following differential amplifier:

- Active amplifiers are ideal \((R_h = \infty, P_{out} = 0)\)
- \(R_1 = 10k\), \(C_1 = 1\text{ nF}\)
- \(R_2 = 1k\), \(C_2 = 1\text{ nF}\)

We want to predict its stability when used in a feedback configuration.

1. Determine the open loop gain, \(A(s)\)
   - Consider stages are at a time
     1. \(A_1(s) = \frac{100}{R_1 + \frac{1}{sC_1}} = \frac{100}{1 + sR_1C_1}\)
     2. \(A_2(s) = \frac{100}{1 + sR_2C_2}\)

   Total transfer function: \(A(s) = A_1(s) \cdot A_2(s) = \frac{10,000}{(1 + sR_1C_1)(1 + sR_2C_2)}\)

   - \(W_{pe} = \frac{1}{R_1C_1} = \frac{1}{(10^5 \cdot 10^{-9})} = 10^5\)
   - \(W_{pe} = \frac{1}{R_2C_2} = \frac{1}{(10^3 \cdot 10^{-9})} = 10^6\)

2. Will this be stable?
   - Depends on feedback factor, \(\beta\)
3. Let's consider different configurations
(a) - Draw in $20 \cdot \log \frac{1}{B} = 20 \cdot \log 1 = 0$ dB line in $|A(s)|$ plot.
- Phase margin = $0^\circ$, unstable, needs compensation.

(b) - Draw in $20 \cdot \log \frac{1}{B} = 20 \cdot \log 100 = 40$ dB line in $|A(s)|$ plot.
- Phase margin = $22^\circ$, stable, but would like a greater phase margin.

For the $\beta = 1$ case, let's compensate to have a phase margin of $45^\circ$.
- Need dotted line on bode plots, move dominant pole to a lower frequency.
- Move $\omega_p$ back to $\omega_p'$.

- Need $\omega_p' = 10^2$.
- Achieve this by introducing additional capacitance at node $Q2$.

\[
\frac{1}{\beta_{1}(s C_1 + C_c)} = \frac{1}{10^4 C_1 + C_c} \Rightarrow C_1 + C_c = 10^{-6} = 1 \mu F
\]
\[
\therefore C_c = 0.999 \mu F
\]

New amplifier:

- New GBW product = $10^2 \cdot 10,000 = 10^6$ (greatly reduced).
- $1 \mu F$ is a huge capacitance to have to add to the circuit.
- If we take advantage of the Miller Effect using the second stage, we could reduce this by a factor proportional to the gain of the second stage.