Stability

In general, both $A$ and $\beta$ depend on frequency.

$$\text{Acc}(s) = \frac{A(s)}{1 + A(s)\beta(s)}$$

- We can assume the loop gain is positive at low frequencies, yielding negative feedback as desired (due to $-1$ in summer input).
- How $A(j\omega)\beta(j\omega)$ behaves at high frequencies will determine the stability.

- Loop gain can be represented by its magnitude and phase: $|A(j\omega)\beta(j\omega)| e^{\phi(j\omega)}$
- At some frequency, the phase will become $180^\circ$, so $e^{\phi(j\omega)} = -1$.
- Now, loop gain $= -|A(j\omega)\beta(j\omega)|$ → positive feedback!!

Three cases:
1. If $|A\beta| < 1$ at this frequency, any oscillations will die out, system will be stable.
2. If $|A\beta| = 1$ at the frequency, Acc = $\infty$ (see equation), circuit will oscillate. (Output present for zero input).
   - In block diagram, remove input, $X_i \sin(\omega t)$, $X_o = -X_i$, signal is sustained.
3. If $|A\beta| > 1$, will have growing oscillations at this frequency, until some non-linear effect drops $|A\beta|$ to 1, at which point the oscillations will be sustained.

Our objective is to find under which conditions oscillations occur, and find ways to avoid them.

Relation Between Stability and Pole Location

- The poles of the amplifier with feedback are determined by the denominator of the Acc expression:
- roots of this equation (called the characteristic equation) are the poles.
- We can plot pole locations in the s-plane:

- Recall that complex poles always come in complex conjugate pairs (need a real output for a real input).
Consider a system with a complex conjugate pole pair at \( s = s_0 \pm j\omega_n \):

1. If any input gets disturbed (impulse function at input), the output will have terms of the form:

\[
V(t) = 2e^{-\alpha t} \cos(\omega_n t)
\]

To find this: \( V(0) = V(0) \cdot H(0) \)

- Impulse function: \( V(0) = 1 \)
- Take inverse Laplace transform to get back to time domain, use some algebra to get equation above.

Consider these cases:

1. \( \sigma_0 < 0 \) (poles in left half plane):

\[
V(0) = 2e^{-\alpha t} \cos(\omega_n t)
\]

- Oscillations are out (exponentially damped)

2. \( \sigma_0 > 0 \) (poles in right half plane):

\[
V(0) = 2e^{-\alpha t} \cos(\omega_n t)
\]

- Oscillations grow (unstable)

3. \( \sigma_0 = 0 \) (poles on imaginary axis):

\[
V(0) = 2 \cos(\omega_n t)
\]

- Sustained oscillations

Both 2 & 3 are unstable, we want 1 to hold true for our system.

How does feedback affect the pole locations?

- Characteristic equation is \( 1 + A_0(s) \beta(s) \)

Exercise three cases: (assume constant \( \beta \))

1. Single pole response: \( A(s) = \frac{A_0}{1 + s/L} \), \( A_0(s) = \frac{A_0(1 + A_0\beta)}{1 + s/L + A_0\beta} \)

Feedback moves pole out to \( \omega_p = \omega_p(1 + A_0\beta) \)

- Final location of pole depends on loop gain \( A_0\beta \).

\[
\omega_p \uparrow \quad s\text{-plane}
\]

Root locus diagram illustrates how pole locations change with feedback (as loop gain \( A\beta \) varies).
2. Two pole response: 

\[ A(s) = \frac{A_0}{(1 + s/w_p_1)(1 + s/w_p_2)} \]

- poles come from \( 1 + A(s) \beta = 0 \) \( \Rightarrow s^2 + s(w_p_1 + w_p_2) + (1 + A_0 \beta)w_p_1 w_p_2 = 0 \)
- can solve quadratic equation for pole locations.

Root Locus:
- unconditionally stable

3. Three pole response:
- no longer guaranteed stability since total phase can exceed -270°

Root Locus:
- at some loop gain, poles cross into RHP, system becomes unstable.

Gain Bode Plot:
- high Q (less stable)
- low Q (more stable)