Differential Pair [Section 3.8]

Basic Differential Pair: just 2 common-source amplifiers with their sources tied together.

- Consider the case when $V_{in} = V_{in}$.
- $V_{os1} = V_{os2} = \frac{2I_b}{2}$.

- Now apply $V_{in} = \frac{V_{id}}{2}$, $V_{in} = -\frac{V_{id}}{2}$ (small)
  - $V_{os1}$ increases by $\frac{V_{id}}{2}$, $V_{os2}$ decreases by $\frac{V_{id}}{2}$
  - More current shifts left to $M_1$.
- How much more? Recall that $g_m = \frac{\Delta I_b}{\Delta V_{os}} = \Rightarrow \Delta I_b = g_m \Delta V_{os}$
  - $\Delta I_{os} = g_m \cdot \frac{V_{id}}{2}$

- For some applications we want linear operation over a large range of differential input voltages.
  - Observe that linear range is wider for low $g_m$ and higher $I_b$.
  - Linear range $= I_b$
  - For these applications, use small ($W/L$) ratio to get small $g_m$ for a given bias current.
Now, let's replace current source by loads from previous schematic to get a realistic implementation.

Current source: Use current mirror.

Loads: Use active loads.

Benefit of active loads over resistive loads:

1. Transistors are much smaller.
2. For high gain, we need large R, with resistors, this leads to large Vdd/dV/ drop forces transistors out of saturation.
3. Provides differential to single ended conversion (with better gain).

\[ V_{out} = \frac{1}{2} \cdot V \cdot g_m \cdot R \]

\[ V_{out} = \frac{V \cdot g_m \cdot R}{2} + \frac{V \cdot g_m \cdot R}{2} \]

(\text{twice as much gain})

There are two types of input signals we can apply to the differential pair:

1. Common-mode: same input is applied to both transistors
2. Differential-mode: the input to one transistor, -ve input to the other.

Both of these types of signals can be applied either as large signal or small signal, let's consider both.

Large Signal Behavior

1. Differential signals - this is the behavior we already discussed.

\[ I_{in} = I_{out} \]

\[ V_{id} \]
Common-mode Signals - relevant for determining range over which diff. pair will still operate properly.

\[ V_{CM_{\text{min}}} \] limited by need to keep \( M_5 \) in saturation.

\[ V_{CM_{\text{max}}} \] limited by need to keep \( M_3 \) & \( M_4 \), \( M_1 \) & \( M_2 \) in saturation.

\[ V_{CM} \text{ at } \Box \text{ is } V_{DD} - V_{BS,3} = V_{DD} - (V_{eff,3} + V_T) \]

\[ = V_{DD} - V_{eff,3} - V_T. \]

\[ \Rightarrow M_1 \text{ needs } V_{BS} = V_{BS} - V_T = V_{eff} \]

\[ \text{max. voltage at } \Box \text{ is } V_A - V_{eff} = V_{DD} - 2V_{eff} - V_T \]

Now, \( V_{CM} \) is 1 \( V_{BS} \) above \( V_C \):

\[ V_{CM_{\text{max}}} = V_{BS} + V_{eff} + V_T = V_{DD} - 2V_{eff} - V_T + V_{eff} + V_T \]

\[ = V_{DD} - V_{eff} \]

\[ \Rightarrow \text{More limited on the low end} \]

Small Signal Behavior

Differential Signals - Both uses "\( T \)" model for analysis.

- alternative to hybrid \( \Pi \) model.\n
- Normally we use

- \( T \) model is equivalent to Hybrid \( \Pi \), use whichever one is most convenient.

- Still no gate current. Since current \( \text{I}_{G} \) dependent current source is same as that in the resistor \( R_S \).
The page contains a detailed derivation of small-signal differential gain in a circuit.

- The text explains how to derive a "quick and dirty" approach to arrive at the same result.
- We have a total of \( i = g_{m} \cdot V_{id} \)
- Flowing into the output node.
- To find \( V_{out} \), we need to know what the output impedance of that node is (small signal).

- \( R_{out} \) will be resistance looking into \( M_{4} \) (current mirror) in parallel with resistance looking into \( M_{2} \).
- Current mirror has \( R_{ds} \cdot R_{ds} \).

Small Signal:
- Looking into drain of \( M_{2} \) and ground to node \( A \).
- From node \( A \), see \( R_{ds} \) in parallel with input resistance of common gate \( M_{1} \) \( \approx \frac{1}{g_{m}} \).

\[
\frac{1}{R_{in}} = \frac{1}{R_{ds} + \frac{1}{g_{m}} || R_{ds}}
\approx \frac{1}{R_{ds} + \frac{1}{g_{m}}} \quad (\frac{1}{g_{m}} << R_{ds})
\approx \frac{1}{R_{ds}} \quad (\frac{1}{g_{m}} << R_{ds})
\]

\[ R_{out} = R_{ds} || R_{ds} \]

Finally, \( V_{out} = i \cdot R_{out} = g_{m} \cdot V_{id} \cdot R_{ds} || R_{ds} \)

**Common-Mode Signal:** Book does not go through this, for a detailed derivation see Sedra & Smith Section 7.5.4.

- We will give an intuitive idea of what is going on, then quote the result.
- When gate voltage of M1, M2 rises by \( V_{CM} \), source voltage rises correspondingly by \( V_{CM} \).
- Current in bias transistor, M5 (thermistor by \( V_{TH} \), split equally between branches.

- Current into node \( \delta \) sees resistance of \( \frac{1}{g_{m3}} \parallel 2\cdot R_{ds} \).

- Induces voltage of \( V_{CM} \left( \frac{1}{g_{m3}} \parallel 2\cdot R_{ds} \right) \).

- This induces current of \( g_{m4} \left( \frac{1}{g_{m3}} \parallel 2\cdot R_{ds} \right) \cdot V_{CM} \) in M4.

- Because quantity above \( \neq 1 \), leads to a net current being driven out of output node, output voltage is this current \( \times \) gain.

\[ A_{CM} = \frac{-1}{2\cdot g_{m3} \cdot R_{ds}} \]

- Usually want low common-mode gain:
- Reject common-mode noise that couples into both inputs.
- From above formula, can reduce \( A_{CM} \) by using bias current source with large output resistance (e.g. cascade current mirror) and by increasing \( g_{m3} \) (use wider devices).

- Measure of how well common-mode noise is rejected is provided by CMRR:

\[ CMRR = \frac{\text{Ad}}{|A_{CM}|} \text{ - very high} \]

\[ A_{CM} \text{ - very low} \]

- Higher = better.

- For above example: \( CMRR = \left( g_{m1} \cdot R_{ds} \parallel R_{ds} \right) \cdot (2 \cdot g_{m3} \cdot R_{ds}) \)

- If all gate resistors are equal: \( = \left( g_{m1} \cdot R_{ds} \right)^2 \)

**CMOS Opamp Design** (Ch. 5 in text)
- We now have all of the building blocks to design a CMOS opamp.
- Basic topology uses multiple gain stages, then an output stage.
Block Diagram:

- Output buffer is only needed when input impedance of the load is small compared to the output resistance of the gain stage.
- How to build this?
  - Stage 1: differential pair with active load (gain)
  - Stage 2: common-source with active load (gain)
  - Stage 3: source-follower (low output impedance)

\[
A_{v1} = g_{m1} \left( \frac{r_{d2}}{r_{d4}} \right) \\
A_{v2} = g_{m5} \left( \frac{r_{d5}}{r_{d6}} \right) \\
A_{v3} = \frac{g_{m7}}{g_{m7} + g_{s7}}
\]

Now, total gain:

\[
A_U = A_{v1} \cdot A_{v2} \cdot A_{v3} \quad \text{(usually on the order of 1000 - 6000)}
\]